Black's Simple Discounting Rule: A Simple Implementation

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Abstract

We propose a simple implementation of Black's (1988) elegant discounting rule. The rule overcomes thorny problems that traditional valuation approaches struggle with, namely identifying the market portfolio, measuring project risk, and assessing the market risk premium. The implementation we propose consists of four steps: (i) finding a benchmark security that correlates with the project's cash flows; (ii) estimating the percentiles of the distribution for which the benchmark return in question equals the risk-free rate; (iii) obtaining information from managers to assess the corresponding percentiles in the cash flow distribution (the so-called conditional mean cash flows); and (iv) discounting those cash flows at the risk-free rate. The evidence suggests that many firms would be able to take these four steps successfully. Computing project and firm value with this approach should be much simpler than with traditional discounted cash flow methods.

1. Introduction

Project value is usually computed by discounting the project's mean net cash flows with an appropriate cost of capital. The capital asset pricing model (CAPM) is typically used to measure that cost (in Graham and Harvey's (2001) survey, 74% of managers claim to always or most always use the CAPM). Some of the problems in doing so include identifying the market portfolio, measuring risk, and computing market risk premiums. Black (1988) has proposed a valuation rule that avoids those problems and can be used under all circumstances in which one can use the CAPM (or the APT). The rule is also relevant in cases in which the CAPM (or the APT) does not necessarily hold. The rule is elegant and simple, but it requires knowledge of the project's future conditional mean net cash flows—conditional on the relevant benchmark return being equal to the risk-free rate. This conditional mean net cash flow is then discounted at the risk-free rate. Estimating conditional mean net cash flows, however, is not straightforward, which has probably dissuaded textbooks from recommending the rule and discouraged practitioners from adopting it—in spite of the fact that estimating unconditional mean net cash flows, as required under the traditional valuation approaches, is in many ways an equally daunting task.

The rule was originally derived by Black (1988) and later analyzed by Long (2000). Discussions are in, among others, Brennan (1995), Myers (1996), and Laitenberger and Löffler (2002). The elegance and simplicity with which Black's rule takes us around the problem of risk-adjustment comes, as we said, at the cost of having to assess conditional mean cash flows. The rule, however, moves the focus of the analyst away from the assessment of discount factors and puts it squarely on the more challenging, and arguably more relevant problem of gauging the project's relevant cash flows. Black's rule would therefore seem to be a simpler tool to compute project (or firm) value than traditional valuation approaches are.

The paper's contribution is to illustrate a simple way to implement Black's rule. We show: (i) how firms can estimate a project's conditional mean net cash flows, the key ingredient in Black's valuation rule; (ii) that our implementation is practicable, that is, it may be used by a sizable number of firms; (iii) that managers generally have the information needed to apply the rule; and (iv) that the details of our implementation are similar across time and countries. The rule and our implementation also apply in cases in which the CAPM (or the APT) does not necessarily hold. What we need is linearity between project NCFs and the return on a benchmark security or index (with an independent error term of zero mean), and efficient markets with respect to the information set of relevance (see also Myers, 1996). We do not propose, however, ways to find that security or index. The gist of the paper is to illustrate how to estimate conditional mean cash flows if the CAPM's assumptions hold.

The implementation we propose consists of four steps: (i) finding a benchmark security or index that correlates with the project's cash flows; (ii) estimating the percentiles of the distribution for which the benchmark return in question equals the risk-free rate (the so-called risk-free percentiles); (iii) obtaining information from managers to assess the corresponding percentiles in the cash flow distribution (the so-called conditional mean cash flows); and (iv) discounting those cash flows at the risk-free rate.

We show that the rule is feasible, in the sense that the necessary benchmark securities seem to exist, the risk-free percentiles can be measured and are reasonably stationary in time and across countries, and most managers claim to have the information it takes. For example, with respect to the existence of a benchmark security, we find that the quarterly net cash flows of a substantial number of firms are significantly correlated with the contemporaneous return on the S&P 500—25% of all the COMPUSTAT firms have an R-squared of about 40% or more.

The rest of the paper is structured as follows. Section 2 derives and discusses Black's rule. Section 3 shows how the rule can be implemented. Section 4 reviews implementation issues and presents evidence indicating that one needs not look farther than the S&P 500 index to find a benchmark index appropriate for a significant number of firms. We also use survey data to argue that many managers have the information required by our implementation. Section 5 examines the empirical properties of what we call risk-free percentiles, the key variable in the implementation we are proposing. We discuss different approaches to estimate those percentiles with historical data, show how they depend on the investment horizon, and conduct international comparisons for a large set of countries. The last section draws conclusions.

2. Black's Discounting Rule

If we want to implement the traditional discounted-cash flow (DCF) rule, we have to solve the following problems: (i) forecasting the project's future net cash flows, assessing their probabilities, and calculating their mean values; (ii) identifying the market portfolio of risky assets; (iii) measuring the market risk premium; (iv) finding the project's beta; (v) estimating the project's risk-adjusted discount rate (if the term structure of interest rates is not flat and the project extends over a number of years, we may need more than one discount rate); and (vi) discounting the forecasted net cash flows with the appropriate risk-adjusted discount rate(s).

Some of these problems are not easy to solve and require substantial guesswork or restrictive assumptions. This is especially the case when it comes to assessing the appropriate risk-adjusted discount rate(s)—according to Fama and French (1997), an almost desperate task.

To get around these problems, Black (1988) proposes an elegant, alternative valuation procedure. What follows provides an intuition for that procedure.

Suppose, for simplicity, that our investment project generates only one net cash flow (NCF) at the end of the year (or at the end of a number of years). Also, suppose there is a security whose return is correlated with that cash flow. Consistent with CAPM assumptions, the security in question could be the market portfolio, but it could also be an industry portfolio or some other security—conceivably, even the firm's own stock. We call this security benchmark security, and the associated return the benchmark return. The NCF can be written as:

$$\tilde{NCF} = \alpha + \beta \times \tilde{R}_M + \tilde{\varepsilon}, \tag{1}$$

where the tilde indicates a random variable, $\tilde{R}_M$ is the arithmetic rate of return (the wealth-relative minus one) on the benchmark portfolio during the year, α is a constant, β is the project's cash-flow beta, and $\tilde{\varepsilon}$ is independent idiosyncratic noise with zero mean.[1] The index therefore captures the cash flow's systematic risk. The error term measures the project's firm-specific or idiosyncratic risk, i.e., possible disturbances in the net cash flow that are unrelated to market-wide events.

Equation (1) tells us that the project's NCF is linearly related to the return on the benchmark portfolio of risky assets. That is, if the project's beta is positive and we ignore the error term, higher benchmark returns lead to higher net cash flows. Moreover, projects with higher cash flow betas react more strongly to changes in benchmark returns—they are riskier.

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[1] If, for example, β = 1,000,000, then, on average, an increase of 100 basis points in $\tilde{R}_M$ results in an increase in NCF of 1,000,000×0.01 = 10,000. The relation between traditional return beta, β_R, and the cash flow's risk-adjusted discount rate, k, is: $\beta = \beta_R \times \frac{E(\tilde{NCF})}{(1+k)}$.

To compute the value of the random NCF in equation (1), it helps to first rearrange equation (1) by writing:

$$N\tilde{C}F = \alpha + \beta \times (\tilde{R}_M - R_F) + \beta \times R_F + \tilde{\varepsilon}, \qquad (2)$$

where R_F is the risk-free rate. Given that we have an unrestricted intercept term, we can rewrite equation (2) with two different beta coefficients. Since it is more general, we focus on that version of equation (1), namely:

$$N\tilde{C}F = \alpha^* + \beta_1 \times (\tilde{R}_M - R_F) + \beta_2 \times R_F + \tilde{\varepsilon}, \qquad (3)$$

The net cash flow is the sum of two random [$\beta_1 \times (\tilde{R}_M - R_F)$ and $\tilde{\varepsilon}$] and two non-random [α^* and $\beta_2 \times R_F$] amounts of money. Its value is therefore the sum of the values of those four terms. Since the two *non-random* quantities are known, we find their value by discounting them at the risk-free rate, namely by calculating

$\frac{\alpha^*}{1 + R_F}$ and $\frac{\beta_2 \times R_F}{1 + R_F}$, respectively.

As for the value of the two *random* amounts of money, it is zero. To show that, we reason as follows. Recognize first that $\tilde{\varepsilon}$ represents pure idiosyncratic risk in the sense that it is independent of $\tilde{R}_M$ and any other market "risk factor" by assumption. It is therefore fully diversifiable and, since its expected value is zero, its present value is zero as well.

The present value of the random amount $\beta_1 \times (\tilde{R}_M - R_F)$ is zero, too. The reason is that you can costlessly construct a replicating portfolio that yields that payoff. To see that, write out this expression as $\beta_1 \times \tilde{R}_M - \beta_1 \times R_F$, and realize that you can replicate that amount of money by simply borrowing the sum β_1 at the risk-free rate and investing it in the benchmark portfolio.[2] In principle, since you have not invested any of your own funds, you should not expect to make any money with this strategy—otherwise, you would have found a money machine. Consequently, the value of the project's NCF equals:

$$\text{Current value of } N\tilde{C}F = \frac{\alpha^*}{1+R_F} + \frac{\beta_2 \times R_F}{1+R_F} = \frac{\alpha^* + \beta_2 \times R_F}{1+R_F}. \tag{4}$$

The quantity $\alpha^* + \beta_2 \times R_F$ in this expression equals the mean net cash flow when the benchmark return equals the risk-free rate—i.e., it equals the mean NCF conditional on that event:

$$E\left[N\tilde{C}F \middle| \tilde{R}_M = R_F\right] = E\left[\alpha^* + \beta_1 \times (\tilde{R}_M - R_F) + \beta_2 \times R_F + \tilde{\varepsilon} \middle| \tilde{R}_M = R_F\right] = \alpha^* + \beta_2 \times R_F, \tag{5}$$

where we use the assumption that the error term, $\tilde{\varepsilon}$, has zero mean and is independent of the benchmark return. The expression $E\left[N\tilde{C}F \middle| \tilde{R}_M = R_F\right]$ is the *conditional* expectation of the net cash flow— $E(N\tilde{C}F) = \alpha + \beta_1 \times E(\tilde{R}_M)$ would be its *unconditional* expectation. Combining equations (4) and (5), we can express the present value of the project's net cash flow as:

$$\text{Current value of } N\tilde{C}F = \frac{\alpha^* + \beta_1 \times R_F}{1+R_F} = \frac{E\left[N\tilde{C}F \middle| \tilde{R}_M = R_F\right]}{1+R_F}. \tag{6}$$

Equation (6) tells us that, to find the current value of a risky net cash flow, all we have to do is discount its conditional expected value at the risk-free rate. That means, we have to

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[2] In reality, the bank will ask for security to cover your liability in case the benchmark return is smaller than the risk-free rate.

measure what the NCF would be on average in the event that the benchmark return equals the risk-free rate, and discount that number at the risk-free rate. "On average" means that we ignore the random term $\tilde{\varepsilon}$. The benchmark return can follow almost any distribution. All we require is that the project cash flow and the benchmark return be linearly related as in equation (1). The conditional mean NCF is the certainty equivalent of the NCF in question. This is Black's discounting rule.

Equation (6) applies also in the case in which projects extend over more than one period. If so, we compute project value by valuing its conditional mean net cash flows separately according to equation (6). The only assumption we make is that NCFs and benchmark returns are linearly related as in equation (1). If project risk gets resolved progressively over time, the benchmark returns are multiperiod benchmark returns computed over the same time horizon as the NCFs in question. However, other patterns of uncertainty resolution are conceivable. For example, suppose the time to a particular project cash flow is N+M months and that none of the current uncertainty about the cash flow is resolved in the first N months. In that case, the appropriate return in equation (1) is the long-period return to a strategy of investing $1 initially in an N-month riskless pure discount bond and then, after N months, investing the proceeds of the bond investment for the remaining M months of the cash flow period in the benchmark security. In the extreme, none of the uncertainty about the cash flow may be resolved until the last month. Examples of this kind of cash flow are monthly profits from an enterprise with a monthly operating cycle where each month's profit is nearly independent of previous months' profits.

Black's discounting rule looks simpler to implement than the traditional DCF rule. If we know the conditional mean NCFs, we can ignore the market risk premium and we don't need to know the project's beta and how it varies over the project's life. As we said, we don't even have to tell what the *market* portfolio is, since the rule applies also in the case of other benchmark portfolios or securities (provided the error term in equation (1) has zero mean and is pure idiosyncratic risk). These are considerable simplifications. Moreover, the rule holds in all situations in which the traditional valuation models such as the CAPM and the APT hold. The simple discounting rule does not work, however, when the NCFs are a non-linear function of the benchmark returns—but neither do the traditional valuation models.[3]

3. Implementing Black's Discounting Rule

The problem in applying Black's rule is the estimation of conditional mean NCFs. As we said, these cash flows are those we observe on average when the return on the benchmark portfolio equals the risk-free rate. Yet it is not clear how we can easily obtain meaningful estimates of those cash flows. One possible solution is to ask managers to tell us what the future net cash flows will be if the benchmark return equals the risk-free rate, on average. Unfortunately, this approach does not seem to be very promising because it is unlikely that managers are consciously aware of that relation.

Myers (1996, p. 99) proposes a two step-forecast. "First, construct scenarios for the business variables corresponding to the macroeconomic conditions implied by a benchmark return equal to the risk-free rate. Then, ask the manager to forecast cash flow for these scenarios. If everything is done consistently, the result should be the conditional forecast Fischer calls for." The problem is translating benchmark returns into macroeconomic conditions. What follows proposes an alternative indirect way to elicit the information we want from managers.

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[3] Options, for instance, are non-linear functions of the benchmark return, since they have positive payoffs above the exercise price and zero payoffs below it. See the discussion in Black (1988), p. 9–10, and Long (2000), p. 10–11.

Figure 1 illustrates what we are after. The histogram on the left shows benchmark returns as we would observe them if we used the CRSP Value Weighted Index as a proxy and they were generated under a normal distribution with the historical parameters estimated for the years 1926–2005 in the U.S.—namely a mean of 9.54% and a standard deviation of 19.51%. In grey, we show the frequency of observations smaller than or equal to an assumed risk-free rate of 3.63% (the historical average annual return on 30-day T-bills).[4] The diagram on the right-hand side of the figure uses these benchmark returns and equation (1) to generate the net cash flows we would expect on a project with an assumed α of 100 and a cash flow beta of 800 (unlike return betas, cash flow betas have values that depend on project size: larger projects tend to have larger cash flow betas). The computation ignores the idiosyncratic risk component—i.e., the $\tilde{\varepsilon}$ term in equation (1). The grey area in the histogram on the right-hand side of the figure is defined by the interval of net cash flows produced by benchmark returns smaller than or equal to the risk-free rate. The conditional mean net cash flow forecast we are interested in is the upper limit of that interval.

A possible heuristic procedure to generate these conditional forecasts is therefore to find the percentile of the distribution the benchmark return defines when it equals the risk-free rate—we are looking for the cumulative density at that point. Because of the monotone increasing relation between net cash flows and benchmark returns, the associated net cash flow will define the same percentile in its own distribution (in other words, the grey areas in the two diagrams of Figure 1 are equal). For example, if the benchmark return equals the risk-free rate at the 20th percentile of its distribution, then the implied net cash flow will also correspond to the 20th percentile of its respective distribution. And once we know the percentile of the net cash flow

[4] Returns are continuously compounded.

distribution we are interested in, we can use managers' cash flow information to identify the NCF that defines that 20^{th} percentile. That's the conditional mean NCF we are looking for. Let us refer to that percentile as the *risk-free* percentile.[5] We are not making any distributional assumptions. The central idea of our implementation approach is that, if idiosyncratic cash flow risk is ignored, the conditional mean cash flow is equal to the cash flow at the risk-free percentile of the cash flow distribution—regardless of return distribution.

We are assuming that the project's cash flow beta is positive. If that beta is negative, meaning that higher benchmark returns induce more negative cash flows, the appropriate conditional mean forecast is the cash flow at the percentile equal to one minus the risk-free percentile.

Another assumption we are making is that, in providing NCF information, managers are intuitively able to abstract from the impact that firm-specific events can have on the cash flows of their projects. In other words, we assume that, in forecasting the possible future project NCFs, they are able to focus on the economy-wide (or industry-wide, if we use an industry index as a benchmark) causes of variation in those cash flows, such as the overall state of the economy, and ignore idiosyncratic accidental, firm-specific events. We come back to this assumption further down.

Conceivably, in trying to assess conditional mean NCFs, it might be easier for managers to break the total NCF for a period into its individual "line-item" components (revenue and expense items). The conditional mean of the total NCF is the sum of the conditional means of those line-item components. Thus, the risk-free percentile method of estimating conditional

[5] The risk-free percentile in the preceding example is 0.20, i.e., we are using the term "risk-free percentile" to refer to the *probability* of a nonpositive excess benchmark return. We refer to the cash flow at the risk-free percentile of the cash flow distribution as the "conditional cash flow."

means can be applied to each line item component separately. The manager could gauge the itemized conditional mean cash inflows and outflows separately, and then discount those estimates at the risk-free interest rate. This approach could be easier (item by item) and more accurate. In particular, (i) it may be easier to distinguish systematic and idiosyncratic sources of variation at the line-item level, and (ii) it may be simpler to estimate the systematic volatility of individual line-item components than the overall systematic volatility (under the normal, you generally need that information to estimate the distribution of the NCFs and its conditional mean). For simplicity, we will ignore this possibility in the following discussion.

Our implementation of Black's rule involves the following four steps: (i) Finding a benchmark index or stock that correlates with the project's cash flows with a pure idiosyncratic error; (ii) estimating the percentiles of the distribution for which the stock return in question equals the risk-free rate; (iii) obtaining information from managers to assess the cash flows that define the same percentiles in the cash flow distribution (i.e., the conditional mean cash flows); and (iv) discounting those conditional mean cash flows at the risk-free rates for the same maturities.

In what follows, we use the CRSP Value Weighted Index as a proxy for the benchmark. Remember, however, that whereas in the implementation of the CAPM we have to look for market portfolio proxies, we do not have to do so here. The next three sections describe steps (ii) to (iv) in the implementation of Black's rule.

3.1. Estimating the Risk-free Percentiles

The following table reports the historical distribution characteristics of the continuously compounded annual stock return on the CRSP Value Weighted Index.[6] As commonly done in the literature, we assume for the moment that this return is normally distributed, even though, as mentioned above, Black's rule (and our implementation) applies also under alternative distributions. In the period of 1942–2005, the average return was 11.39% and the standard deviation was 15.58%. The table also shows the annual (continuously compounded) yields-to-maturity on Treasury securities with maturities between 1 and 5 years during the same time period.[7] We use those yields as proxies for both the historical and the current risk-free rate.

	CRSP Value Weighted Index	Treasury yields				
	1942–2005	1 year	2 years	3 years	4 years	5 years
Average	11.39%	5.13%	5.24%	5.32%	5.39%	5.47%
Standard deviation	15.58%					

Suppose we have an investment project whose net cash flows are linearly related to the benchmark return as in equation (1). Assuming the distribution of benchmark returns is expected to remain the same over time, we can use the numbers in the table to assess our risk-free percentiles. Since investment projects can last several years, we assume that equation (1) holds with benchmark returns measured over a different number of years, corresponding to the time horizon of the project's NCFs. Consequently, the net cash flow two years ahead will be related

[6] The empirical analysis uses continuously compounded returns. One can always express arithmetic returns as equivalent continuously compounded returns. This expedient makes it is easier to estimate multiperiod mean returns and return variances in our subsequent calculations—over T periods, the mean return equals the one-period mean multiplied by T, and the return variance equals the one-period variance multiplied by T. See Fama (1996) for a similar analysis.

[7] Since we don't have Treasury yields for 3- and 4-year maturities, we compute them as linear interpolations of the available 2- and 5-year yields.

to the benchmark return over the next two years, the net cash flow three years ahead to the benchmark return over the next three years, etc. As pointed out above, this assumes that NCF uncertainty gets resolved progressively over time. Other resolution patterns are conceivable and consistent with equation (1).

The table below uses the historical data to calculate the benchmark return's average and standard deviation as well as the relevant risk-free rate for time horizons of one to five years. For example, the average benchmark return over a three-year horizon is 34.17%, the benchmark return's standard deviation is 26.99%,[8] and the risk-free rate is 15.96%.[9] These values imply that the percentile of the distribution for which the benchmark return equals the risk-free rate over a three-year horizon is 24.99%.[10] The table shows that the risk-free percentile falls from 34.39% for a time horizon of one year to 19.78% for a horizon of five. The reason for the decline is that the mean return increases faster with the investment horizon than the return dispersion does—the mean return increases linearly with T whereas the standard deviation of the return increases with the square root of T.

Year of NCF	Cumulative average R_M	Standard deviation of R_M	Cumulative risk-free rate	Percentile for which R_M equals or is smaller than the risk-free rate
1	11.39%	15.58%	5.13%	34.39%
2	22.78%	22.03%	10.48%	28.83%
3	34.17%	26.99%	15.96%	24.99%
4	45.56%	31.16%	21.56%	22.06%
5	56.95%	34.84%	27.35%	19.78%

[8] Given continuous compounding, and safe for rounding errors, the cumulative average return equals three times the annual average (34.17% = 3×11.39%). The associated standard deviation equals the square root of three times the annual standard deviation (26.99% = √3×15.58%). See Fama (1996) for similar computations and tables.

[9] Given continuous compounding, that average yield equals three times the annualized three-year yield, namely 15.96% (= 3×5.32%).

[10] That percentile is computed by first setting the cumulative three-year stock return equal to the cumulative three-year risk-free rate and then standardizing the result with the cumulative average three-year stock return and its standard deviation. The standard normal variable in question equals (15.96–34.17)/26.99 = –0.6748 and the associated normal distribution is 24.99%.

3.2. Estimating the Distribution of Future Net Cash Flows

The second step is estimating the distribution of future net cash flows.[11] For logical convenience, given the discussion in the preceding section, we assume a Gaussian distribution. Most managers do not know the distribution of future NCFs in much detail. They know aspects of it, however. And, under normality, all we need is two points on that distribution.[12] For example, they might have an idea about the mean of that distribution and an estimate of the probability that the cash flows will fall under a certain value.[13] Alternatively, they might be able to state mean values for various scenarios, such as a pessimistic and an optimistic one (actually, these are truncated means of the overall net-cash-flow distribution). And at the same time, they might have a rough idea of the probability with which the cash flows will fall under the average value under the pessimistic scenario, or exceed the average value under the optimistic one. We can use that information to pinpoint the full distribution of the future cash flows of an investment project.

3.3. Estimating the Conditional Mean Cash Flows

The third step involves quantifying the conditional mean cash flows of our project. Given the information gathered in the two preceding sections, we can do so fairly easily.

3.4. Discounting the Conditional Mean Cash Flows at the Risk-free Rate

Computing the value of conditional mean cash flows simply requires discounting them at the corresponding risk-free rates.

[11] This section relies on input by a team of students (Heinz Brägger, Thomas Himmel, Roman Käser, Andreas Nauer, and Jürg Rippl) in the Rochester-Bern Executive M.B.A. Program who implemented Black's rule for an actual investment project.

[12] In fact, the same applies under any two-parameter distribution with real-line support.

[13] Similar information is needed in other contexts to state value-at-risk or cash-flow-at-risk measures.

3.5. A Comprehensive Example

Having discussed the individual steps to implement Black's rule, we can illustrate this valuation approach with an example.

A producer of plastic products is thinking of replacing one of its extrusion machines. The new machine costs 1.2 million. It requires less energy, and it is faster and more reliable than the current one. Upkeep and maintenance costs are about the same as for the old machine. The relevant horizon is five years. How could one use Black's rule to help the producer decide?

Suppose we use the CRSP Value Weighted Index as the benchmark security and the historical annual yields-to-maturity on Treasury securities with maturities between 1 and 5 years as measures of the risk-free rate. Unless otherwise indicated, all data are historical. On the basis of these assumptions, we know from the preceding discussion that the relevant risk-free percentiles are as follows.

Year of NCF	Cumulative average R_M	Standard deviation of R_M	Cumulative risk-free rate	Risk-free percentile
1	11.39%	15.58%	5.13%	34.39%
2	22.78%	22.03%	10.48%	28.83%
3	34.17%	26.99%	15.96%	24.99%
4	45.56%	31.16%	21.56%	22.06%
5	56.95%	34.84%	27.35%	19.78%

To assess the distribution of future NCFs, let's assume the project manager has given us the following data (all NCFs in thousands).

Year of NCF	Average NCF State of market: Pessimistic	Probability of lower NCF	Average NCF State of market: Normal	Probability of lower NCF
1	200	10%	500	50%
2	300	10%	700	50%
3	300	10%	700	50%
4	200	10%	500	50%
5	100	10%	200	50%

The manager is able to state the unconditional average future NCFs. Hence, we only need to estimate the standard deviation of the future NCFs to identify their distributions. Once we have those estimates, we can calculate the conditional mean NCFs we are searching for. The following table summarizes the resulting calculations (NCFs in thousands). Note that the risk-free rates are the current as opposed to the historical ones.

Year of NCF	Estimated mean of future NCF distribution	Estimated standard deviation of future NCF distribution	Risk-free percentile	Estimated conditional mean NCF	Current risk-free rate (continuously compounded)
1	500	234.09	34.39%	405.93	5.25%
2	700	312.12	28.83%	525.73	5.30%
3	700	312.12	24.99%	489.38	5.45%
4	500	234.09	22.06%	319.71	5.50%
5	200	78.03	19.78%	133.71	5.60%

For example, to estimate the standard deviation of the future NCF in year 4, we write:

$$\Phi\left(\frac{200-\mu}{\sigma}\right)=\Phi\left(\frac{200-500}{\sigma}\right)=0.1, \text{ which implies } \frac{200-500}{\sigma}=-1.282 \text{ and } \sigma=234.09.$$

Similarly, to assess the conditional mean NCF in year 4, we write:

$$\Phi\left(\frac{NCF_C-\mu}{\sigma}\right)=\Phi\left(\frac{NCF_C-500}{234.09}\right)=0.2206.$$

Upon inverting this expression, we obtain:

$$\frac{NCF_C-500}{234.09}=-0.77017 \text{ and therefore } NCF_C=-0.77017\times 234.09+500=319.71.$$

The information in the table can be used to compute project value. All we have to do is discount the conditional mean NCFs with the appropriate annualized risk-free rates.

$$\begin{aligned} NPV &= -1,200+405.93\times e^{-0.0525}+525.73\times e^{-0.053\times 2}+489.38\times e^{-0.0545\times 3}+319.71\times e^{-0.055\times 4}+ \\ &\quad +133.71\times e^{-0.056\times 5} \\ &= 431.22. \end{aligned}$$

The value of the project is 431.22 thousand. Based on this point estimate, buying the machine appears to be financially attractive.

4. Implementation Issues

Our approach faces three major challenges, namely whether we can find a benchmark security with returns closely correlated with a project's NCFs, whether managers are able to ignore idiosyncratic sources of NCF volatility, and whether managers have the information necessary to estimate the distribution of future NCFs. Let's begin with the first question.

4.1. Coefficients of Determination of Equation (3)

The question is whether we can find a benchmark security that correlates closely with the project's cash flows, as postulated by equation (3). What we are interested in is the size of the correlation between a cash flow that will be realized at time t (e.g., a year from now) and a benchmark asset return that will also be realized at time t. The relevant joint distribution of the cash flow and the asset return is the distribution *conditional* on information available at time t–1. Thus, the relevant correlation is the correlation between the asset return and the cash flow forecast error defined as the time t cash flow minus the time t–1 forecast of the cash flow. To measure that correlation, we can treat equation (3) as a regression model.

If we focus on one-year-ahead cash flows, then the regression to estimate is the regression of one-year-ahead forecast errors on a benchmark return realized at the same time as the forecast error. The accounting literature suggests that a good time-series model for quarterly earnings is a seasonal random walk in which the forecast of n-th quarter earnings next year are the n-th quarter earnings this year (see, for instance, Bernard and Thomas (1990)). We therefore replace NCF_t in regression equation (3) with $NCF_t - NCF_{t\text{-}4Q}$, where Q stands for quarter. For two- and three-year investment horizons, the innovation in NCF_t is the difference between the NCF_t in quarter t and the NCF_t in the same quarter two respectively three years before (NCF_{t-8Q} and NCF_{t-12Q}).

The S&P 500 is our benchmark index. The risk-free rate is the return on the constant-maturity Treasury series obtained from the CRSP Government Bond Files for each particular investment horizon. The sample comprises all Compustat firms, excluding financials. We measure NCFs with quarterly net cash flows from operations as reported in Compustat data item #108.[14] To control for possible nonstationarity, we estimate our regressions also by scaling the quarterly net cash flows with the total assets reported at the beginning of that given quarter (Compustat data item #44).[15] We use 5- and, alternatively, 10-year sample estimation periods. The 5-year window covers the years 2001 to 2006; the 10-year period includes the years 1997 to 2006. Firms are excluded from the 5- (10-) year sample if they have fewer than 20 (30) observations.

Table I presents the estimation results. For each individual firm, we run regressions for either window. Moreover, each regression is estimated alternatively with actual and standardized NCFs. The table reports sample moments of the R-squared distribution yielded by each regression specification, namely average, first and third quartile, 90th percentile, and maximum value. The top half of the table displays the results based on the 5-year estimation

[14] For a robustness check, we also measure NCFs following the approach in Minton and Schrand (1999) as sales (Compustat data item #2) less cost of goods sold (item #30), less selling, general, and administrative expenses (item #1), less the change in net working capital. Net working capital is the sum of non-missing amounts for accounts receivable (item #37), inventory (item #38), and other current assets (item #39) minus the sum of non-missing amounts for accounts payable (item #46), income taxed payable (item #47), and other current liabilities (item #48). Our results, however, don't change significantly.

[15] Our results are unaffected when we scale NCFs with the value of property, plant, and equipment (PP&E) instead of the value of total assets.

window; the bottom half exhibits them for the 10-year estimation window. In each half, we show our estimates for 1-, 2-, and 3-year returns, respectively.

Let us focus on the results for the 5-year estimation window first. The average R-squared of the regression is between 0.2 and 0.3. The strongest correlations are observed for longer investment horizons and unstandardized NCFs. For the regressions with those characteristics and one-year returns, for example, the average R-squared is 0.22; for three-year returns it is 0.29. The third quartiles of the distributions and, especially, their 90th percentiles, yield fairly sizable R-squared values. In particular, the third quartile of the distribution for the regressions with unstandardized NCFs and one-year returns is 0.32, the 90th percentile is 0.44 (the maximum is 0.81). These numbers go up further when returns are measured over 2 and 3 years. With three-year returns, the corresponding results are 0.44 and 0.60, respectively (the maximum is 0.89). The explanatory power of the model generally declines when we use an estimation window of 10 years. It is possible that the risk characteristics of aggregate operating NCFs change over time, which would increase the error term in the regressions.

There are consequently a number of firms for which Black's rule seems to apply. If we take a look at the 90th percentile of the one-year regressions we just described, there are about 320 firms (=0.1×3,206) with an R-squared larger than 0.44. This observation applies to the first-year NCFs of a hypothetical project. For the NCFs in years 2 and 3, the panel shows, as we saw, even larger R^2s. The set-up of our investigation probably makes our quest for large explanatory power difficult, since we are considering a large benchmark aggregate and, especially, company-wide NCFs. Aggregate NCFs could represent the consolidation of widely different projects with diverse risk characteristics. Conceivably, breaking down the benchmark returns to industry (possibly firm-specific) returns, and focusing on project (as opposed to company-wide) NCFs, could yield even tighter fits.

A potential problem is autoregressive residuals. To examine its severity, we compute a Breusch-Godfrey test of serial correlation of order one. Table II reports our findings. The average p-value across the 3,206 sample firms is 0.179 for one-year horizons. Hence, there does not seem to be a problem, on average. The average p-value increases with the investment horizon, being equal to 0.268 for 2-year and 0.295 for 3-year returns. Hence, the autoregression problem, if any, tends to go away as we extend the investment horizon, on average. There are, however, a number of individual firms for which serial correlation is more palpable. Expressed as a fraction of total firms in the sample, the number of firms with p-values smaller than 5 percent equals 49 percent (=(3,101–1,992)/3,101) in the regressions with one-year returns. That fraction drops as we extend the investment horizon, yet the number of firms with significant autoregressive residuals remains fairly large—1,109 (=3,101–1,992) with 2-year returns, and 855 (=2,785–1,930) with 3-year returns. The serial correlation is comparatively more sizable when we use a 10-year estimation window.

Even though serial correlation is not a serious problem, on average, we have to find out whether it is more critical in the case of the more promising firms in our investigation, namely those with a better fit. If so, the encouraging results of Table I would have to be questioned.

Table III explores that issue by estimating equation (3) for firms with uncorrelated residuals based on the Breusch-Godfrey test. Correlation is defined by p-values smaller than 5 percent. The layout of the table is the same as that of Table I. For simplicity, we focus again on the coefficients of determination obtained using the 5-year estimation window and without standardizing the NCFs. There is no evidence that firms with serially uncorrelated residuals have lower R^2s. If anything, the R^2s are slightly larger. For example, the average is now 0.258, compared with 0.222 in the unconditional sample in Table I; similarly, the 90^{th} percentile is 0.485 here, compared with 0.437 there. Consequently, serial correlation does not seem to cast doubt on our finding that equation (3) is appropriate for a fairly large number of firms.

4.2. Ignoring idiosyncratic sources of NCF variation

The second serious challenge our implementation faces is the assessment of a project's future *unconditional* mean cash flows and of the associated standard deviation (or of any two statistics of the NCF distribution, under the assumption of normality we are making). Combining these two pieces of information, we can assess the distribution of future NCFs and identify the conditional mean NCFs we need. Yet to come up with the information in question, managers have to be able to disregard firm-specific events (i.e., the disturbance factor $\tilde{\varepsilon}$ in equations (1) to (3)). We simply assumed managers have that ability without much explanation. It would seem that paying no attention to firm-specific occurrences is quite a natural inclination. It would be very difficult for managers to forecast NCFs based on speculations concerning the occurrence of fortuitous events such as secretarial mistakes or accidents in the company's plants—an almost unlimited set of possible occurrences.

There are, however, two arguments that help us make our case more formally. The first is that idiosyncratic events cancel each other out over time. Hence, managers with long enough working experience should have learned to focus on systematic events almost automatically. The second argument is that many executives familiar with risk management practices are consciously able to distinguish company-specific events from economy- or industry-wide changes, since those two classes of events have different policy implications. Adverse firm-specific events can be prevented by establishing appropriate internal guidelines and codes of conduct. In contrast, there is little an importer of Japanese high-tech equipment can do to prevent market-wide events such as a hike in the value of the Japanese Yen.

Of course, even if managers have not learned to disregard the idiosyncratic sources of cash-flow variation, the project analyst can always help them do so with the proper instructions. It is important to recognize, however, that the problem of ignoring firm-specific considerations is not limited to our implementation of Black's rule. It confronts also the user of the traditional DCF methods. In fact, one could argue that the task is simpler under Black's rule. The rule does not require a market portfolio but rather a security or a tracking portfolio correlated with the project's NCF that yields an uncorrelated error term with zero mean. If so, to distinguish between systematic and idiosyncratic developments one can follow the rule that whatever is not company-specific is systematic. That rule fails, however, in the case of the CAPM, where systematic means market-wide. Not all events that are not company-specific are market-wide—the demise of the buggy-whip industry in the early 20th century as a result of the expansion of the automobile industry would seem to be an example. We have to ignore these disturbances if we want to implement the CAPM. Yet we do not have to do that if we use an industry index (or an individual benchmark security) under Black's rule.

4.3. What Managers Seem to Know

Assuming they can ignore idiosyncratic disturbances, the ultimate question is whether managers possess the information necessary to assess the distribution of future NCFs. To find out, we surveyed all the alumni of the Rochester-Bern Executive MBA program. These managers have all been exposed to DCF methods and to basic statistical concepts and techniques. They should therefore be able to provide an informed opinion about the practicability of our implementation. In a questionnaire, we asked those who have been involved in computations of (medium/large) project or firm value with a DCF approach the following question: "We would like to know whether [in your past computations] you would have been able to provide any of the following information for the years for which you estimated cash flows (this generic term refers to net cash flows, free cash flows, or residual cash flows). Please understand that we are simply trying to find out what information, if any, is commonly available—we are by no means suggesting that one does or should know the information below." We then gave them the following list of characteristics of the hypothetical distribution of future NCFs to choose from:

A-I	The average cash flow;
A-II	The standard deviation of the cash flow;
B-I	A break-even cash flow (i.e., the minimal cash flow necessary to make the project worthwhile);
B-II	A rough probability of observing the break-even cash flow;
C-I	The pessimistic cash flow;
C-II	A rough probability of observing the pessimistic cash flow;
D-I	The optimistic cash flow;
D-II	A rough probability of observing the optimistic cash flow;
E	A rough probability of observing a zero cash flow.

To implement Black's rule under the normality assumption, we require at least two points on the hypothetical distribution of project cash flows. Consequently, we need any two of the items: A-I, A-II, B-I & II, C-I & II, D-I & II, and E.

We sent the questionnaire to 496 managers; 212 (42.7%) filled it out. Of those, 125 (59%) were recently involved in valuation—virtually all with a DCF approach. Table IV reports the number and percentage of respondents able to quantify individual items in the preceding list in the context of their projects. About three out of every four respondents could have stated average, break-even, or pessimistic cash flows. Substantially fewer individuals would have been capable to provide measures of dispersion or probabilities. Specifically, about 36% could have indicated the probability of observing the pessimistic, the optimistic, or the break-even cash flows; twenty-three percent could have quantified a rough probability of observing zero cash flows, and 18% had an estimate of the standard deviation of those cash flows.

Taken together, about half of the managers (63 out of 125) who claim to use a DCF valuation approach would have been able to implement Black's discounting rule along the lines we are suggesting. In fact, this figure is probably downward biased since 54.4% of all responding managers would have found it easier to provide information about the statistics A-I to C-II for the cash flows' individual "line-item" components than for the sum of those components.

5. Empirical Characteristics of Risk-free Percentiles

In order to implement Black's rule, we have to compute risk-free percentiles. In the discussion above, we relied on U.S. data from the years 1942–2005 to do so. Yet the use of data in the comparatively far past makes sense only if the percentiles in question are reasonably stationary over time. This section examines that issue for different sub-periods throughout the years of 1926–2005. Furthermore, we assess the magnitude and behavior of those percentiles over different investment horizons, and investigate various estimation approaches. Finally, we ask how risk-free percentiles compare across capital markets—in integrated markets, we would expect similar values.

5.1. Risk-free Percentiles for One-month Investment Horizons: Historical U.S. Estimates

Data for our computations are from the monthly CRSP files for individual decades in the period 1926–2005. The CRSP Value-Weighted Index is our benchmark security and the 30-day T-bill rate is the proxy for the risk-free rate. For simplicity, we work with excess returns, defined as the difference between benchmark returns and contemporaneous risk-free interest rates. The risk-free percentile for an investment horizon of one month is therefore the cumulative probability of a monthly excess return equal to or smaller than zero.

Table V reports distribution characteristics. The first two columns of the table display the mean and the variance of the monthly excess returns in each decade. The third column shows the risk-free percentile in each decade under the normal approximation using the estimated mean and variance for the decade in question. The risk-free percentiles go from 38.7% (1946–1955) to 51.9% (1966–1975), although most of the observations are between 42% and 46%. For the full 1926–2005 period, the estimate is 46.4%.

To assess whether these risk-free percentiles are stationary, we use a Wilcoxon rank-sum (Mann-Whitney) test and compare the distribution of monthly excess returns in each decade with that of the full 1926–2005 period (exclusive of the decade under consideration). The z-statistics of this test are reported in column (4). They suggest that the distributions of the monthly excess returns in each individual decade do not differ significantly from the distribution for the full period at customary levels of significance. Hence, the distribution of excess returns does not seem to change significantly over time, which suggests that the risk-free percentiles are stationary. A reasonable estimate of the mean risk-free percentile for one-month investment horizons is therefore the 46.4% figure obtained for the overall 1926–2005 period. The exception in our test is the 1966–1975 decade, which differs significantly from the overall distribution with confidence 0.95. We should note, however, that observing a significant difference for one decade out of eight is not very surprising.

Column (3) assumes normality. The Shapiro-Wilk test in column (5), however, rejects that assumption for most decades. To assess the importance of the deviation from normality with regard to the estimation of the risk-free percentiles, column (6) computes the risk-free percentiles on the basis of the actual distributions of excess returns (the so-called exact method). These estimates are almost always smaller than those obtained under the normality assumption. The deviation between the two, however, is relatively contained. For the full period, for example, the exact risk-free percentile is 40.1%, compared with 46.4% under the normal. Moreover, the average deviation is 5.35% across decades (not shown).

The last column in the table illustrates the exact binomial 95%-confidence intervals for the exact risk-free percentiles in each decade. The risk-free percentile of 40.1% measured for the full period is inside the binomial confidence intervals in each individual decade—the exception is 1966–1975. This implies stationarity of the sample distributions of excess returns, consistent with the conclusions implied by the Wilcoxon rank-sum test.

5.2. Risk-free Percentiles for One-year Investment Horizons: Historical U.S. Estimates

The preceding table looks at monthly risk-free percentiles. In practice, however, we are interested in longer investment horizons. Table VI therefore extends the hypothetical horizon to one year and estimates the associated risk-free percentiles. The data are still those for the U.S. Panel A reports annualized figures. Column (1) shows mean annualized excess returns; to annualize, we multiply the average monthly excess return stated in Table V for a particular decade times 12. Column (2) computes the variance of the annualized excess return for each decade by multiplying the corresponding variance of the monthly excess return in Table V times

12.[16] For the full period, for example, we find a mean annualized excess return of 5.9% and a variance of 3.6%. Column (3) then uses each decade's parameter estimates to compute risk-free percentiles under the normal. These annualized percentiles are generally lower than the monthly percentiles reported in the preceding table. The reason is that, in annualizing, the mean increases with 12 and the standard deviation increases with the square root of 12. The estimated percentiles go from 16.0% (1946–1955) to 56.6% (1966–1975), although most of the observations are between 30% and 48%. For the full 1926–2005 period, the estimate is 37.7%, which is not much different from the average of 35.3% we can compute across decades (not shown).

For a comparison, Panel B of the table performs the calculations with historical annual (as opposed to annualized) excess return data. That computation yields an overall risk-free percentile of 38.2% under the assumption of normality, a figure that is almost identical to the annualized 37.7% found in Panel A. Observed annual excess returns are not Gaussian according to a Shapiro-Wilk test either, but the deviation is considerably less extreme than that observed with monthly data—the z-statistics are 9.801 with monthly and 2.071 with annual data.

As with monthly data, the risk-free percentiles obtained under the normal are larger than the exact risk-free percentiles (38.2% vs. 32.5%, respectively). If true, this difference would imply a bias in the conditional cash flow estimate equal to 15-16% of the standard deviation of the systematic part of the cash flow.[17] We also find that the 38.2% estimate falls inside the exact

[16] There are alternative ways of forecasting volatility. One could use GARCH models on a rolling basis. Figlewski (1997), however, shows that GARCH(1,1) volatility forecasts for the S&P 500 with monthly data are worse than forecasts based on historical estimates—and they are generally not better with daily data. Hence, the use of historical volatility estimates for forecasting purposes is both simple and relatively precise.

[17] The percentile difference of 5.7% (=38.2%–32.5%) implies a difference in the systematic part of the cash flow of 15.4% under a standard normal distribution.

binomial 95%-confidence interval shown in the panel. Hence, we cannot reject the hypothesis of no difference between the estimated and the actual risk-free percentiles.

Overall, the evidence suggests that we can assess risk-free percentiles for one-year investment horizons equally well by annualizing monthly data or by using annual data directly. Moreover, both estimates are reasonably close to the actual risk-free percentile observed for the full sample period. With annual data, the normality assumption seems to be more acceptable than it is with monthly data. Remember, however, that normality is not a necessary assumption, even though it can simplify the estimation of risk-free percentiles, especially over longer investment horizons. We could therefore also use the exact risk-free percentiles as a proxy for the risk-free percentiles.

One potential problem in annualizing monthly excess returns is the implicit assumption of serial independence. What follows uses U.S. data to examine how reasonable that assumption is. Table VII reports autocorrelation coefficients of monthly excess returns for individual decades during the years of 1926–2005. According to the calculations, no serial correlation coefficient is numerically large or significant at the first six lags in any individual decade. The only significant coefficients are those computed for the overall period. They are, however, numerically fairly minuscule. As it turns out, allowing for serially correlated monthly excess returns yields risk-free percentile estimates almost identical to those obtained in Table VI under the assumption of zero autocorrelation (not shown).

5.3. Risk-free Percentiles for Different Maturities: Historical U.S. Estimates

Having shown that we can compute annualized risk-free percentiles ignoring serial correlation, we estimate risk-free percentiles for longer investment horizons. Table VIII displays the results for maturities up to ten years under the assumption of normality. For our computations, we use the annualized excess-return average and variance estimates reported in Panel A of Table VI for the full sample period of 1926–2005 (5.911% and 3.569%, respectively). For an investment horizon of T years, the average excess return is T×5.911% and the variance T×3.569%. For a four-year horizon, for example, the average excess return is 23.6% (=5.911%×4) and the return variance 14.3% (=3.569%×4). These parameters imply a risk-free percentile of 26.6%. The risk-free percentiles we obtain go from 37.7% for a one-year horizon down to 16.1% for a ten-year horizon.

5.4. Risk-free Percentiles across International Capital Markets

The last step in our analysis performs an international comparison of risk-free percentiles. The countries of interest are Australia, Canada, France, Germany, Hong Kong, Japan, Spain, Switzerland, and the U.K. With the U.S., these are the largest stock markets in 2006 according to the World Federation of Exchanges. Table IX computes risk-free percentiles for one-year investment horizons for these countries under four different approaches.

Column (1) estimates risk-free percentiles using annualized monthly excess returns under the assumption of normality. Column (2) reports risk-free percentiles obtained from the estimated means and variances of the historical annual excess returns in each country under the assumption of normality. Column (3) exhibits exact risk-free percentiles based on the historical distributions of annual excess returns in each country. The last column presents the exact binomial 95%-confidence intervals for the risk-free percentiles in each country.

If capital markets were reasonably integrated, their risk-free percentiles ought to be similar. The evidence supports that conjecture. Regardless whether we estimate them by annualizing monthly returns or using historical annual returns under the assumption of normality, the estimates are reasonably similar. For the most part, these estimates are also fairly close to the exact risk-free percentiles—at the very least, the exact binomial test fails to reject equality. As a result, the cross-country average risk-free percentiles are almost the same no matter how we estimate them—38.2% when we annualize monthly excess returns, 39.3 when we use historical annual returns under the normal, and 36.5% when we simply compute the exact percentiles.

6. Conclusions

Black's (1988) rule gets around a number of estimation problems that face the analyst trying to implement traditional DCF valuation approaches. Among other things, he does not have to identify the market portfolio; all he needs to do is find a security or an index that is correlated with the project's cash flows with an error term that is pure noise. Moreover, he does not have to measure the project's risk or its changes. And he does not have to assess the market risk premium.

The rule can be used under all circumstances in which one can use the standard valuation approaches, including the CAPM and the APT. Moreover, it can be used in situations under which the CAPM and the APT do not apply. The rule looks fairly simple, but it requires knowledge of the project's conditional mean cash flows (conditional on zero excess returns). Estimating those conditional cash flows, however, is not straightforward, which has probably dissuaded textbooks from recommending the rule and discouraged practitioners from adopting it. This paper illustrates a simple way to estimate a project's conditional mean cash flows.

Our approach involves four steps: (i) Finding a benchmark security that correlates with the project's NCF and yields an error term that captures pure idiosyncratic risk; (ii) estimating the percentiles of the distribution for which the stock return in question equals the risk-free rate over different investment horizons; (iii) obtaining information from managers to assess the cash flows that define the same percentile in the corresponding cash flow distributions (the conditional mean cash flows required by Black's rule); and (iv) discounting the conditional mean cash flows at the yields-to-maturity on riskless pure discount bonds with the same maturity as the cash flows. Much of these steps can be programmed fairly easily in a spreadsheet. Since value is additive, we can also decompose a cash flow into component cash flows and value the components separately using, if appropriate, different benchmark securities.

To assess the existence of the required benchmark security, we estimate regressions of the operating NCFs of COMPUSTAT firms against the contemporaneous return on the S&P 500. Even at that high level of aggregation, we can find a fairly strong correlation for a number of firms. Hence, there are reasons to believe that Black's rule is implementable for a large number of companies.

As for the risk-free percentiles, one can estimate them from past excess returns on the benchmark security. The easiest way is to use annual data and to assume normality. Given these percentiles, one has to find ways to elicit information concerning the distribution of future project NCFs from managers. Under two-parameter distributions such as the normal, all one needs is two statistics (e.g., mean and variance) of that distribution. A survey of managers suggests that a majority would be able to provide that information.

It would seem that our implementation of Black's rule is simpler and more accurate than the traditional valuation approach is. At the very least, it is a valid complement. The most difficult task is probably step (iii) above. However, many managers in our survey claim to have the necessary information. In contrast, the traditional valuation approach requires the forecast of future unconditional mean NCFs. That is probably equally difficult. Moreover, as we said, the traditional approach calls for risk-adjusted discount rates. As it turns out, that estimation is almost hopeless under the best of conditions—so much so, according to Fama and French (1997), that "two of the ubiquitous tools in capital budgeting are a wing and a prayer" (p. 179).

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Figure 1
Benchmark returns and associated conditional mean NCFs

The diagrams use the continuously compounded return on CRSP’s Value Weighted Index as the benchmark return. That return is assumed to be normally distributed with parameter values equal to those observed during 1926–2005, namely an annual average of 9.54% and a standard deviation of 19.51%. The continuously compounded risk-free rate is 3.63%. The investment project is assumed to have a NCF with an α-value of 100 and a cash flow beta of 800.

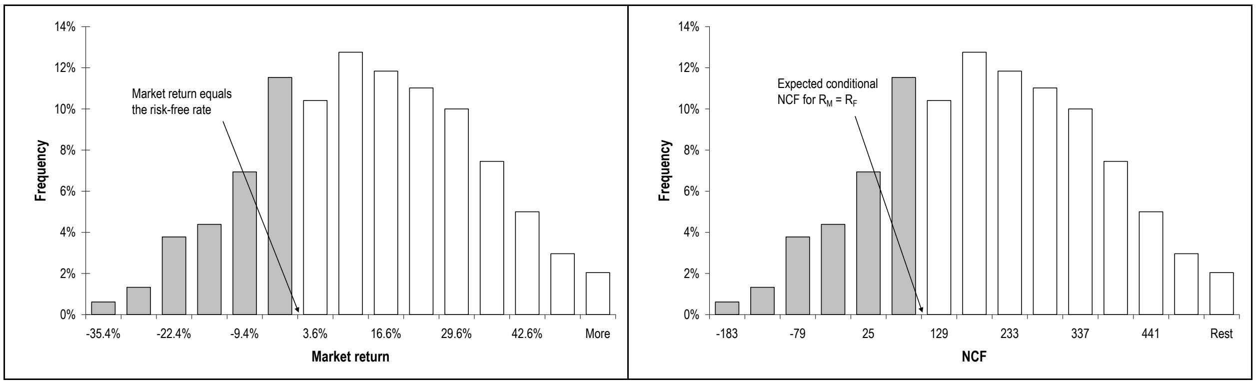

Table I
Correlation between net cash flows and contemporaneous returns on the S&P 500

The table shows R-squared of OLS regressions of the innovation in a company's quarterly net cash flows on benchmark returns according to equation (3) in the text:

$$NCF_t - NCF_{t-4Q} = \alpha + \beta_1 \left(R_t - R_{f,t} \right) + \beta_2 R_{f,t} + \varepsilon_t .$$

For one-year investment horizons, the innovation in net cash flows is the difference between the net cash flow in quarter t and the net cash flow in the same quarter of the prior year. R_t is the benchmark return and $R_{f,t}$ is the risk-free rate. Returns are one-, two-, and three-year returns, respectively. For two- and three-year investment horizons, the innovation in NCF is the difference between the NCF in quarter t and the NCF in the same quarter two (NCF_{t-8Q}), respectively three (NCF_{t-12Q}) years before. We use the S&P 500 as our benchmark index. The risk-free rate is the return on the constant-maturity Treasury series obtained from the CRSP Government Bond Files. The sample comprises all Compustat firms, excluding financials. We measure NCFs with quarterly net cash flows from operations as reported in Compustat data item #108. Alternatively, we scale these net cash flows with total assets (TA, Compustat data item #44). The data are collected on a quarterly basis. We use 5- and 10-year sample periods for our regressions. The 5-year window includes the years 2001 to 2006; the 10-year period covers the years 1997 to 2006. Firms are excluded from the 5- or the 10-year sample periods if they have less than 20 and 30 observations, respectively. Q1 and Q3 in the tables are the first and third quartile of the sample distribution; P90 is the 90th percentile.

Returns	NCF definition	R-squared of regressions					
		Average	Q1	Q3	P90	Max	Obs.
5-year window (2001-2006)							
1-year return	NCF	0.222	0.096	0.322	0.437	0.805	3,206
	NCF/TA	0.227	0.094	0.340	0.453	0.807	3,191
2-year return	NCF	0.282	0.113	0.423	0.576	0.834	3,101
	NCF/TA	0.291	0.121	0.437	0.588	0.843	3,092
3-year return	NCF	0.293	0.121	0.439	0.601	0.890	2,785
	NCF/TA	0.291	0.120	0.435	0.598	0.900	2,779
10-year window (1997-2006)							
1-year return	NCF	0.098	0.027	0.139	0.224	0.643	3,936
	NCF/TA	0.101	0.029	0.143	0.234	0.668	3,915
2-year return	NCF	0.199	0.072	0.293	0.426	0.800	3,755
	NCF/TA	0.189	0.064	0.279	0.414	0.775	3,735
3-year return	NCF	0.266	0.112	0.393	0.536	0.836	3,461
	NCF/TA	0.245	0.098	0.363	0.509	0.846	3,443

Table II
Serial correlation in the residuals of regressing NCF on contemporaneous benchmark returns

The table shows test statistics for a Breusch-Godfrey test of serial correlation of order one for regressions of the innovation in a company's quarterly net cash flows on benchmark returns. The results refer equation (3) in the text:

$$NCF_t - NCF_{t-4Q} = \alpha + \beta_1 \left(R_t - R_{f,t} \right) + \beta_2 R_{f,t} + \varepsilon_t .$$

For one-year investment horizons, the innovation in net cash flows is the difference between the net cash flow in quarter t and the net cash flow in the same quarter of the prior year. R_t is the benchmark return and $R_{f,t}$ is the risk-free rate. Returns are one-, two-, and three-year returns, respectively. For two- and three-year investment horizons, the innovation in NCF is the difference between the NCF in quarter t and the NCF in the same quarter two (NCF_{t-8Q}), respectively three (NCF_{t-12Q}) years before. We use the S&P 500 as our benchmark index. The risk-free rate is the return on the constant-maturity Treasury series obtained from the CRSP Government Bond Files. The sample comprises all Compustat firms, excluding financials. We measure NCFs with quarterly net cash flows from operations as reported in Compustat data item #108. Alternatively, we scale these net cash flows with total assets (TA, Compustat data item #44). The data are collected on a quarterly basis. We use 5- and 10-year sample periods for our regressions. The 5-year window includes the years 2001 to 2006; the 10-year period covers the years 1997 to 2006. Firms are excluded from the 5- or the 10-year sample periods if they have less than 20 and 30 observations, respectively.

Returns	NCF definition	Test statistics for a Breusch-Godfrey test of serial correlation			
		Average Chi^2	Average p-value	Number of regressions with p-value > 5%	Number of regressions
5-year window (2001-2006)					
1-year return	NCF	4.124	0.179	1,644	3,206
	NCF/TA	3.903	0.196	1,741	3,191
2-year return	NCF	3.441	0.268	1,992	3,101
	NCF/TA	3.073	0.286	2,119	3,092
3-year return	NCF	2.852	0.295	1,930	2,785
	NCF/TA	2.627	0.309	2,019	2,779
10-year window (1997-2006)					
1-year return	NCF	10.400	0.048	578	3,936
	NCF/TA	9.432	0.057	711	3,915
2-year return	NCF	9.289	0.084	915	3,755
	NCF/TA	8.628	0.089	1,002	3,735
3-year return	NCF	7.408	0.126	1,222	3,461
	NCF/TA	6.896	0.133	1,275	3,443

Table III
Correlation between net cash flows and S&P 500 returns for firms with insignificant serial correlation in their residuals

The table shows R-squared of regressions of the innovation in a company's quarterly net cash flows on benchmark returns. We report results only for those regressions in Table II for which a Breusch-Godfrey test of serial correlation of order one is insignificant at the 5%-level. The results refer specification (3) in the text:

$$NCF_t - NCF_{t-4Q} = \alpha + \beta_1\left(R_t - R_{f,t}\right) + \beta_2 R_{f,t} + \varepsilon_t .$$

For one-year investment horizons, the innovation in net cash flows is the difference between the net cash flow in quarter t and the net cash flow in the same quarter of the prior year. R_t is the benchmark return and $R_{f,t}$ is the risk-free rate. Returns are one-, two-, and three-year returns, respectively. For two- and three-year investment horizons, the innovation in NCF is the difference between the NCF in quarter t and the NCF in the same quarter two (NCF_{t-8Q}), respectively three (NCF_{t-12Q}) years before. We use the S&P 500 as our benchmark index. The risk-free rate is the return on the constant-maturity Treasury series obtained from the CRSP Government Bond Files. The sample comprises all Compustat firms, excluding financials. We measure NCFs with quarterly net cash flows from operations as reported in Compustat data item #108. Alternatively, we scale these net cash flows with total assets (TA, Compustat data item #44). The data are collected on a quarterly basis. We use 5- and 10-year sample periods for our regressions. The 5-year window includes the years 2001 to 2006; the 10-year period covers the years 1997 to 2006. Firms are excluded from the 5- or the 10-year sample periods if they have less than 20 and 30 observations, respectively. Q1 and Q3 in the tables are the first and third quartile of the sample distribution; P90 is the 90th percentile.

Returns	NCF definition	R-squared of regressions					
		Average	Q1	Q3	P90	Max	Obs.
5-year window (2001-2006)							
1-year return	NCF	0.258	0.123	0.374	0.485	0.805	1,644
	NCF/TA	0.266	0.124	0.388	0.491	0.807	1,741
2-year return	NCF	0.319	0.142	0.478	0.614	0.834	1,992
	NCF/TA	0.325	0.149	0.485	0.622	0.843	2,119
3-year return	NCF	0.316	0.136	0.470	0.625	0.890	1,930
	NCF/TA	0.312	0.135	0.475	0.625	0.900	2,019
10-year window (1997-2006)							
1-year return	NCF	0.127	0.031	0.177	0.329	0.643	578
	NCF/TA	0.130	0.036	0.186	0.317	0.668	711
2-year return	NCF	0.259	0.096	0.394	0.548	0.800	915
	NCF/TA	0.246	0.091	0.372	0.523	0.775	1,002
3-year return	NCF	0.323	0.152	0.474	0.608	0.836	1,222
	NCF/TA	0.302	0.131	0.453	0.588	0.846	1,275

Table IV
Survey results

We surveyed the alumni of the Rochester-Bern Executive MBA to find out whether they would have the information necessary for our implementation of Black's discounting rule. We asked those who have been involved in computations of (medium/large) project or firm value with a discounted cash flow (DCF) approach whether they would have been able to quantify the items listed below. The table reports numbers and percentages of respondents for each individual item. We sent the questionnaire to 496 managers in January of 2007. 212 (42.7%) filled out the questionnaire and returned it by mid March 2007. Of the respondents, 125 (59%) were recently involved in valuation (the overwhelming majority with a DCF approach). Our implementation of Black's discounting rule requires at least two points on the hypothetical distribution of project cash flows. Consequently, it calls for any two of the following items: A-I, A-II, B-I & II, C-I & II, D-I & II, and E.

Ability to provide the following items of the cash flow distribution		Number of respondents	Percentage of respondents
A-I	The average cash flow	107	85.6%
A-II	The standard deviation of the cash flow	23	18.4%
B-I	A break-even cash flow (i.e., the minimal cash flow necessary to make the project worthwhile)	90	72.0%
B-II	A rough probability of observing the break-even cash flow	44	35.2%
C-I	The pessimistic cash flow	94	75.2%
C-II	A rough probability of observing the pessimistic cash flow	44	35.2%
D-I	The optimistic cash flow	94	75.2%
D-II	A rough probability of observing the optimistic cash flow	44	35.2%
E	A rough probability of observing a zero cash flow	28	22.4%
F	Rather than for the aggregate cash flow, would it have been easier to provide information about A-I to C-II for its individual "line-item" components (i.e., revenues, variable costs, investments, etc.)?	68	54.4%
Ability to provide any two of A-I, A-II, B-I & II, C-I & II, D-I & II, and E		63	50.4%

Table V
Monthly risk-free percentiles in the U.S.

The table estimates risk-free percentiles on the basis of continuously compounded monthly return data from the U.S. for individual decades in 1926–2005. The CRSP Value Weighted Index is the benchmark security and the yields on 30-day T-bills are the proxies for the risk-free interest rate. The individual columns report: (1) The mean monthly excess return, defined as the difference between the benchmark return and the contemporaneous risk-free rate; (2) The variance of the monthly excess return; (3) The z-statistic of a Wilcoxon rank-sum (Mann-Whitney) test of the hypothesis that the sub-period samples come from the same distribution as the overall 1926–2005 sample (excluding the sub-period in question); (4) The fraction of monthly excess returns less than or equal to zero (i.e., the risk-free percentile) in a given decade under the normal approximation. In this computation, the normal distribution has parameter values equal to the mean and the variance of the excess return reported for the decade in question; (5) The z-statistic of a Shapiro-Wilk test of normality; (6) The exact risk-free percentile, defined as the proportion of observed monthly excess returns smaller than or equal to zero in any given decade; (7) The binomial exact 95%-confidence intervals for that percentile; (8) The number of monthly returns in each decade. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively (two-sided tests).

Period	Mean excess return	Excess return variance	Risk-free percentile, assuming normality	Wilcoxon rank-sum test (z-statistics)	Shapiro-Wilk tests for normality (z-statistics)	Exact risk-free percentile	Exact binomial 95%-confidence intervals	Number of observa-tions
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1926–1935	0.172%	0.912%	49.3%	–0.021	4.002***	42.5%	[33.5%, 51.8%]	120
1936–1945	0.702%	0.390%	45.5%	–1.001	5.006***	35.8%	[27.3%, 45.1%]	120
1946–1955	1.065%	0.137%	38.7%	–1.242	1.425*	37.5%	[28.8%, 46.8%]	120
1956–1965	0.669%	0.116%	42.2%	–0.137	3.123***	34.2%	[25.8%, 43.4%]	120
1966–1975	–0.230%	0.229%	51.9%	2.210**	0.709	50.8%**	[41.6%, 60.1%]	120
1976–1985	0.477%	0.188%	45.6%	0.707	0.930	45.8%	[36.7%, 55.2%]	120
1986–1995	0.636%	0.193%	44.2%	–0.292	5.645***	35.0%	[26.5%, 44.2%]	120
1996–2005	0.450%	0.222%	46.2%	–0.223	3.360***	39.2%	[30.4%, 48.5%]	120
1926–2005	0.493%	0.297%	46.4%	N/A	9.801***	40.1%	[37.0%, 43.3%]	960

Table VI
Risk-free percentiles for one-year investment horizons in the U.S.

The table estimates risk-free percentiles for one-year investment horizons with continuously compounded monthly return data from the U.S. for the individual decades of 1926–2005. The CRSP Value Weighted Index is the benchmark security and the yields on 30-day T-bills are the proxies for the risk-free interest rate. Panel A reports annualized figures. Column (1) shows the mean annualized excess return, defined as the difference between the return on the CRSP Value Weighted Index and the contemporaneous risk-free rate; to annualize, we multiply the average monthly excess return, as reported in Table 2 for the decade in question, times 12. Column (2) computes the variance of the annualized excess return by multiplying the variance of the monthly excess return reported in Table 2 for the decade in question times 12. Column (3) reports the implied fraction of annualized excess returns less than or equal to zero (the risk-free percentile) in a given decade. Panel B gives descriptive statistics concerning actual annual excess returns. ** denotes significance at the 5% level (two-sided tests).

Panel A: Annualized risk-free percentiles for individual decades in 1926–2005

Period	Mean annualized excess return	Annualized excess return variance	Risk-free percentiles, assuming normality
	(1)	(2)	(3)
1926–1935	2.066%	10.948%	47.5%
1936–1945	8.422%	4.682%	34.9%
1946–1955	12.774%	1.644%	16.0%
1956–1965	8.024%	1.386%	24.8%
1966–1975	–2.760%	2.744%	56.6%
1976–1985	5.728%	2.250%	35.1%
1986–1995	7.626%	2.313%	30.8%
1996–2005	5.405%	2.667%	37.0%
1926–2005	5.911%	3.569%	37.7%

Panel B: Annual risk-free percentiles for individual decades in 1926–2005

Average annual excess return	5.911%
Variance of annual excess return	3.900%
Implied risk-free percentile (fraction of excess returns ≤ 0)	38.2%
Shapiro-Wilk tests for normality, z-statistic	2.071**
Exact risk-free percentile (proportion of monthly excess returns ≤ 0)	32.5%
Exact binomial 95%-confidence interval	[22.4%, 43.9%]

Table VII
Autocorrelation coefficients of monthly excess returns in the U.S.

The table reports autocorrelation coefficients of monthly excess returns for individual decades during the 1926–2005 period. The computations rely on continuously compounded monthly return data from U.S. for the years 1926–2005. The CRSP Value Weighted Index is the benchmark security and the yields on 30-day T-bills are the proxies for the risk-free interest rate. The null hypothesis for the Portmanteau test is that the data are white noise. *** denotes significance at the 1% level.

Period	Autocorrelation						Portmanteau test for white noise (12 lags)	
	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Portmanteau (Q) statistic	p-value
1926–1935	0.223	–0.073	–0.239	–0.002	0.085	–0.007	26.863	0.008
1936–1945	–0.111	0.170	–0.071	0.063	0.090	0.040	13.801	0.314
1946–1955	0.016	0.045	–0.051	–0.036	0.118	–0.001	5.716	0.930
1956–1965	0.140	–0.075	0.044	0.179	–0.004	–0.122	19.341	0.081
1966–1975	0.113	–0.001	0.071	0.067	0.056	–0.058	7.705	0.808
1976–1985	0.031	–0.085	–0.081	0.041	0.231	–0.023	11.392	0.496
1986–1995	0.055	–0.063	–0.097	–0.185	0.039	–0.022	12.340	0.419
1996–2005	0.039	–0.066	0.014	–0.080	0.004	0.109	6.148	0.908
1926–2005	0.100***	–0.012***	–0.099***	0.009***	0.083***	–0.003***	32.725	0.001

Table VIII
Risk-free percentiles for different investment horizons in the U.S.

The table reports risk-free percentiles for different cash-flow maturities (investment horizons) of a hypothetical project. The computations rely on continuously compounded monthly return data from U.S. for the years 1926–2005. The CRSP Value Weighted Index is the benchmark security and the yields on 30-day T-bills are the proxies for the risk-free interest rate. The various columns show: (1) The investment horizon, i.e., the maturity of the cash flows of the hypothetical investment project; (2) The estimated mean cumulative excess returns for the different investment horizons (= 5.911% × investment horizon; 5.911% is the mean annualized excess returns from Panel A of Table VI); (3) The variance of the excess returns (= 3.569% × investment horizon; 3.569% is variance of the annualized excess returns from Panel A of Table VI); (4) The risk-free percentiles assuming normality.

Investment horizon in years	Estimated mean cumulative excess return	Estimated excess return variance	Risk-free percentiles, assuming normality
(1)	(2)	(3)	(4)
1	5.911%	3.569%	37.7%
2	11.822%	7.138%	32.9%
3	17.733%	10.707%	29.4%
4	23.644%	14.276%	26.6%
5	29.555%	17.845%	24.2%
10	59.110%	35.690%	16.1%

Table IX
Risk-free percentiles for one-year investment horizons; an international perspective

This table computes risk-free percentiles for one-year investment horizons based on annualized and annual excess returns for one-year investment horizons across countries. To compute mean annualized excess returns, we multiply the average monthly excess return observed for each country's sample period times 12. The variance of the annualized excess return is calculated as the variance of the monthly excess return for each country's sample period times 12. Risk-free percentiles are then computed under the assumption of normality (column 1). Column (2) calculates risk-free percentiles under the assumption of normality using the mean and variance of the (historical) annual excess returns in each country. Column (3) exhibits exact risk-free percentiles based on the (historical) annual excess returns in each country. Column (4) shows exact binomial 95%-confidence intervals for the risk-free percentiles in each country.

Countries	Period	Annualized excess returns	Annual excess returns		
		Implied risk-free percentiles	Risk-free percentiles under the normal	Exact risk-free percentiles	Exact binomial 95%-confidence intervals
		(1)	(2)	(3)	(4)
Australia	1926–2005	33.9%	35.7%	28.8%	[19.2%, 40.0%]
Canada	1934–2005	36.5%	36.9%	36.1%	[25.1%, 48.3%]
France	1926–2005	37.5%	40.0%	43.8%	[32.7%, 55.3%]
Germany	1926–2005	44.2%	40.2%	37.5%	[26.9%, 49.0%]
Hong Kong	1970–2005	37.3%	38.9%	33.3%	[18.6%, 51.0%]
Japan	1926–2005	35.8%	37.5%	41.3%	[30.4%, 52.8%]
Spain	1941–2005	37.6%	40.0%	35.4%	[23.9%, 48.2%]
Switzerland	1966–2005	39.1%	41.8%	32.5%	[18.6%, 49.1%]
U.K.	1926–2005	38.3%	40.4%	36.3%	[25.8%, 47.8%]
U.S.	1926–2005	37.7%	38.2%	32.5%	[22.4%, 43.9%]
Average	–	38.22%	39.32%	36.52%	–