Measuring Performance in a Dynamic World: Conditional Mean-Variance Fundamentals

Abstract

We develop conditional alpha performance measures that are consistent with conditional mean-variance analysis and the magnitude and sign of the implied true conditional time-varying alphas. The sequence of conditional alphas and betas are estimable from surprisingly simple unconditional regressions. Other common performance measures are derived from the conditional investment opportunity set based on its conditional asset return moments. Our bootstrap analysis of Morningstar mutual fund returns data demonstrates that the differences between existing conditional alpha measures and our proposed alpha are substantive for typical parameterizations.

Keywords: Conditional Performance Measurement, Conditional Mean-Variance Analysis, Conditional Jensen's Alpha, Conditional Sharpe Ratio, and Conditional Appraisal Ratio

JEL Classification: D82, G11, G12, G14, and G23

1. Introduction

Performance measurement is central to finance theory in assessing asset pricing models as well as in finance practice when determining a fund manager's ability. We propose and empirically examine a conditional version of Jensen's (1968) alpha. The use of "unconditional" measures of performance in a dynamic world with time varying conditional asset return moments suffers from a number of limitations. In particular, unconditional performance measures fail to identify the states in which superior or poor performance is obtained. Investors and fund managers will be interested in the states in which positive and negative performance occurs, especially if these states are predictable.

Our goal is to create a conditional alpha specification that is computable from a conditional mean-variance investment opportunity set under standard econometric assumptions. We posit a linear relationship between instruments and asset returns and use conditional moment specifications to derive the resultant conditional regression coefficients and implied conditional alpha. In addition, we provide an unconditional regression specification that provides an equivalent conditional alpha and a convenient estimation approach. We begin our analysis in parameter space thereby avoiding any inference errors or data issues. Our conditional asset return moment specification follows, among others, Campbell (1987), Harvey (1989), or Shanken (1990). Our approach differs from the extant literature that models conditional regression coefficients as linear functions of available instruments.[1] Instead, we treat conditional parameters as derived directly from the relevant conditional moments. The differences between our

[1] Korkie and Turtle (2002) provide a related discussion of the unconditional alphas in Fama and French (1995).

conditional regression parameters and ad hoc linear specifications that linearly relate instruments and coefficients may be substantial. Using bootstrap and simulation analysis, we demonstrate that the conditional alphas from the extant literature are not equivalent to the conditional Jensen alphas implied by the conditional investment opportunity sets.

The remainder of the paper is organized as follows. In the next three sections, we present a linear framework where errors are conditionally distributed as multivariate normal homoskedastic random variables. In Section 2, we develop our specification of the conditional investment opportunity set. We retain the linkage between the multivariate return moments that describe the conditional investment opportunity set and underlying information instruments that define the conditional return moments. Section 3 derives the conditional alpha for an informed manager. The proposed measure and the development allow our results to be nested within the seminal literature of Jensen (1968), Sharpe (1966), and Treynor and Black (1973). In Section 4, we demonstrate how a simple unconditional regression may be employed to estimate conditional alphas. We also compare our conditional alpha with other conditional alphas in the literature. Section 5 presents our empirical results using 341 mutual funds and our five instrument information set. Using a bootstrap analysis, we provide finite sample properties of our proposed measure and the literature's alpha estimates. We show that the proposed conditional alpha differs substantially from the literature's alternative measures.[2] In Section 6, we provide simulation results for our approach and the related approaches in

[2] Our results may also help to explain the puzzle presented by Ahn, Cao, and Chrétien (2003) in an unpublished working paper. Using a methodology similar to Cochrane and Saá-Requejo (2000), they conclude that, among others, the Ferson and Schadt (1996) conditional alphas produced a substantial number of inadmissible mutual fund performance measures with a tendency to overestimate performance. Admissibility is determined by whether the fund's performance falls outside zero arbitrage bounds. The observed empirical poor performance of these measures in practice may be due to estimation problems or to model problems.

the literature using various sample sizes and error distributions. We also relate the conditional alpha to the conditional Sharpe measure and other traditional performance measures. Section 7 offers concluding comments.

2. Conditional Moments and the Conditional Investment Opportunity Set (CIOS)

We are interested in examining the conditional performance of an active manager in the context of a conditional investment opportunity set. Consider a market of N risky assets and a riskless asset providing investment opportunities of passive portfolios constructible from the $N+1$ assets, over a forthcoming period. The investment opportunities are defined by the excess return means and variances of all possible buy and hold portfolios over the forthcoming period, *t*. At the beginning of the period, the mean and covariance moments of the N assets are known parameters conditional on information instruments. For ease of notation and for expositional convenience, we consider the information instruments, $\left\{Z_{k,t}\right\}$, $k=1,2,...,N_Z$, observed at the beginning of the period *t*.

The N assets' conditional mean excess returns, $\mu_{t|Z_t} = f\left(Z_t\right)$, are functionally related to the N_Z instruments in the information set at the beginning of the period and the period *t* asset excess returns may be written as

$$R_{t|Z_t} = \mu_{t|Z_t} + \eta_t \,, \qquad (1)$$

where $\eta_t \sim G\left(0, \Sigma_{\eta\eta|Z_t}\right)$ is the $(N \times 1)$ random error with $(N \times N)$ conditional covariance matrix, $\Sigma_{\eta\eta|Z_t}$, and G is a normal distribution.[3, 4]

For example, a typically employed linear regression instrument model (see for example, Campbell, 1987) is

$$R_{t|Z_t} = a_t + b_t Z_t + \eta_t \,, \tag{2}$$

where, if we assume constant coefficients, then

$$b = \Sigma_{RZ} \Sigma_{ZZ}^{-1} \,,$$

$$a = \mu - \Sigma_{RZ} \Sigma_{ZZ}^{-1} \mu_Z \,,$$

μ is the $(N \times 1)$ vector of unconditional mean excess returns,

Σ_{RZ} is the $(N \times N_Z)$ matrix of covariances between asset excess returns and instrument values, and

Σ_{ZZ}^{-1} is the $(N_Z \times N_Z)$ inverse of the covariance matrix of instrument values.

The implied conditional excess return mean vector is

$$\mu_{|Z} = a + bZ \tag{2a}$$

and the conditional error covariance matrix in this regression example is

$$\Sigma_{|Z} = \Sigma - \Sigma_{RZ} \Sigma_{ZZ}^{-1} \Sigma_{RZ}' \,, \tag{2b}$$

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[3] The instruments may be in indicator variable form or continuous variable form.

[4] Although the multivariate conditional and unconditional distributions for period t excess returns have known mean and covariance parameters in this analysis, the distributions need not necessarily be normal nor the instrument model assumed to be linear. The conditional mean vector, $\mu_{|Z}$, and conditional covariance matrix, $\Sigma_{|Z}$, are sufficient to define the conditional investment opportunity set (CIOS) of risky assets, regardless of how the matrices are specified. We discuss the generic distribution extension in section 6.

where Σ is the $(N \times N)$ unconditional excess return covariance matrix.

The forward-looking CIOS, implied by a set of information instruments, defines the conditional excess return means and standard deviations of all possible passive portfolios of the N assets. Efficient portfolios are those combining the riskless asset and the risky asset tangency portfolio, or equivalently, its properly weighted portfolio constituents. The investment opportunity set has well-known properties that apply irrespective of whether it is based on unconditional or conditional moments (Roll, 1977). These properties include the alpha or security market line deviations that arise when an index is inefficient, as discussed in the following section. There is a sequence of CIOSs that evolve through time conditional on the sequence of instruments' values.

In the next section, CIOS properties are used to define the conditional alphas for actively managed portfolios.

3. The Performance Alpha Implied by a Conditional Investment Opportunity Set

In the preceding market, suppose that N_{p1} portfolios, constructible from N constituent assets and the riskless asset, span the efficient mean-variance investment opportunities of all passive portfolios.[5] In this case, all passive portfolios will exhibit zero (abnormal) performance relative to the $N_{p1}+1$ portfolios, as required (Chen and Knez (1996)). The simplest representation is $N_{p1}=1$, where this portfolio is the passive tangency portfolio of the N assets.[6] While the underlying theory does not specify the

[5] Some literature has called these intersecting portfolios, because their efficient set intersects the point of tangency of the riskless asset and all risky assets (cf., MacKinlay, 1987).

[6] In a single index capital asset pricing context, this is the market index.

number of spanning portfolios that are required, Lehmann and Modest (1988) suggest that in a multifactor model five factors are adequate.

In addition to the N_{p1} passive spanning portfolios, suppose that there are now N_{p2} mutual funds each actively managing the N assets from the beginning to the end of the single period, where $N_p = N_{p1} + N_{p2}$. There are two effects of adding active funds to the passive assets investment opportunity set. First, enlarging the asset set to include actively managed assets will enlarge the CIOS. Second, the funds may trade on richer information, resulting in a change in the CIOS due to different conditioning instruments' values. The overall effect is that the formerly optimal passive tangency portfolio is inefficient and the funds will exhibit parametrically good or bad conditional performance conditioned on the richer information.

Because the CIOS is defined by conditional matrices that remove the effect of the instruments from both the funds and the spanning portfolios, the $\left(N_{p2} \times 1\right)$ vector of conditional alphas is

$$\alpha_{2|Z} = \mu_{2|Z} - \beta_{2|Z}\mu_{1|Z} \tag{3a}$$

where

$\mu_{2|Z}$ is the $\left(N_{p2} \times 1\right)$ conditional mean vector and

$\mu_{1|Z}$ is the $\left(N_{p1} \times 1\right)$ conditional mean vector of the spanning portfolios.

The $\left(N_{p2} \times N_{p1}\right)$ matrix of conditional betas is given by

$$\beta_{2|Z} = \Sigma_{21\,|Z}\Sigma_{11\,|Z}^{-1} \tag{3b}$$

where

$\Sigma_{21|Z}$ is the $\left(N_{p2} \times N_{p1}\right)$ conditional error covariance matrix of the active funds with the N_{p1} spanning portfolios and

$\Sigma_{11|Z}$ is the $\left(N_{p1} \times N_{p1}\right)$ conditional error covariance matrix of the spanning portfolios, where $\Sigma_{21|Z}$ and $\Sigma_{11|Z}$, the conditional covariance matrices, are partitions of the full conditional covariance matrix $\Sigma_{|Z}$ from (2b).

There are two important points implicit in the performance measure given by (3a). First, the conditional alpha is calculated from the conditional return moments. Second, the conditional alpha in (3a) is equivalent to a multivariate regression intercept obtained using conditional returns and moments and given by

$$R_{2|Z} = \alpha_{2|Z} + \beta_{2|Z} R_{1|Z} + e_{2|Z} , \tag{4}$$

where $\alpha_{2|Z}$ and $\beta_{2|Z}$ are given by (3a) and (3b) respectively and the $\left\{Z_{k,t}\right\}$, $k = 1,2,...,N_Z$ are known and fixed quantities for the single period, *t*.

The conditional covariance matrices, from (2b), are

$$\Sigma_{11|Z} = \Sigma_{11} - \Sigma_{1Z} \Sigma_{ZZ}^{-1} \Sigma_{1Z}' \tag{4a}$$

and

$$\Sigma_{21|Z} = \Sigma_{21} - \Sigma_{2Z} \Sigma_{ZZ}^{-1} \Sigma_{1Z}' , \tag{4b}$$

where Σ_{11} is the $\left(N_{p1} \times N_{p1}\right)$ unconditional covariance matrix, Σ_{1Z} is the $\left(N_{p1} \times N_z\right)$ matrix of unconditional covariances between the spanning portfolios and the instruments, Σ_{21} is the $\left(N_{p2} \times N_{p1}\right)$ matrix of unconditional covariances between the spanning portfolios and funds, and Σ_{2Z} is the $\left(N_{p2} \times N_z\right)$ matrix of unconditional covariances between the fund portfolios and the instruments.

It is clear from equation (3a) that the active portfolio's conditional alphas are dependent on the information instruments because they affect conditional return moments. An empirical specification for the conditional moments may be used to construct the conditional moments and to calculate the performance measures of interest. A simple two-step process for obtaining an actively managed fund's alpha, conditional on information known at the beginning of the period, might proceed as follows. First, the conditional means and conditional covariances of the fund and the spanning portfolios are specified based on rational forecasts of their returns and squared return errors.[7] Second, the alphas are computed from (3a) using the conditional means and conditional covariances and the specific instrument values, $\left\{Z_{k,t}\right\}$, $k = 1,2,...,N_Z$. The process may be repeated sequentially to obtain a fund's sequence of conditional alphas for multiple periods. Alternatively, the conditional alphas can be obtained from the unconditional moments and the conditioning information, as discussed next.

The Conditional Alpha and Unconditional Moments

The $\left(N_{p2} \times 1\right)$ vector of alphas from (3a) can be expressed in terms of unconditional rather than conditional moments by substituting the expressions for the conditional moments. For a linear instrument model, the conditional means are from (2a),

$$\mu_{2|Z} = a_2 + b_2 Z = \mu_2 + b_2 z \qquad (5a)$$

[7] Rationality is defined here as "minimum rationality", Rubinstein (2001).

and

$$\mu_{1|Z} = a_1 + b_1 Z = \mu_1 + b_1 z \,, \tag{5b}$$

where $z = (Z - \mu_Z)$ is the $(N_Z \times 1)$ random vector of instrument deviations from the means and b_1 is an $(N_{p2} \times N_Z)$ coefficient matrix.

Substituting (4a), (4b), (5a) and (5b) in (3a) results in the following expression for the conditional alphas,

$$\alpha_{2|z} = (\mu_2 + b_2 z) - \left[\Sigma_{21} - \Sigma_{2z}\Sigma_{zz}^{-1}\Sigma_{z1}\right]\left[\Sigma_{11} - \Sigma_{1z}\Sigma_{zz}^{-1}\Sigma_{z1}\right]^{-1}(\mu_1 + b_1 z) \tag{6}$$

where $b_1 = \Sigma_{1z}\Sigma_{zz}^{-1}$ and $b_2 = \Sigma_{2z}\Sigma_{zz}^{-1}$. The conditional alphas may thus be obtained by substituting the appropriate unconditional moments into (6). The expression may then be evaluated at specific instrument values, $\hat{z} = (\hat{Z} - \mu_z)$, for any given period.[8]

In subsequent sections, we provide another very simple alternative regression procedure for the conditional alphas and estimate them for a representative sample of active mutual funds. We also compare our results with alternative estimation procedures in the literature.

[8] Equation (6) may also be rewritten to show the relationship between the conditional alphas and the traditional unconditional alphas, betas, instrument regression coefficients, along with the conditional and unconditional covariance matrices. In particular, the conditional alpha is linear in the instruments,

$$\alpha_{2|z} = (\mu_2 + b_2 z) - \left[\Sigma_{21} - \Sigma_{2z}\Sigma_{zz}^{-1}\Sigma_{z1}\right]\left\{\Sigma_{11}^{-1} + \Sigma_{11}^{-1}\Sigma_{1z}\left[\Sigma_{zz} - \Sigma_{z1}\Sigma_{11}^{-1}\Sigma_{1z}\right]^{-1}\Sigma_{z1}\Sigma_{11}^{-1}\right\}(\mu_1 + b_1 z)$$

$$= \alpha_2 + b_2 z - \beta_2 b_1 z - A\mu_1 - Ab_1 z$$

where $A = \beta_2\Sigma_{1z}\Sigma_{zz|1}^{-1}\beta_z - b_2\Sigma_{z1}\Sigma_{11|z}^{-1}$, $\beta_z = \Sigma_{z1}\Sigma_{11}^{-1}$, and $\Sigma_{zz|1} = \Sigma_{zz} - \Sigma_{z1}\Sigma_{11}^{-1}\Sigma_{1z} = \Sigma_{zz} - \beta_z\Sigma_{1z}$.

This result makes use of the identity $[A + BDC]^{-1} = A^{-1} - A^{-1}B\left[CA^{-1}B + D^{-1}\right]^{-1}CA^{-1}$ for nonsingular $[A + BDC]$ to evaluate $\Sigma_{11|z}^{-1}$ (cf., Judge et al, 1985, p. 947).

4. Unconditional Regressions and Conditional Alphas

As described in equation (6) of the previous section, the conditional alpha may be written and estimated as a complex function of unconditional return moments. Alternatively, if more complex nonlinear relations between the conditional moments and the instruments are posited, equation (3a) may be used to determine the conditional alpha. This may suggest that estimation of the conditional alpha using unconditional regression moments might be difficult without restrictions on the coefficient estimators or other onerous assumptions.

In this section, we develop an alternative and simple multivariate regression model to define conditional performance measures given a linear instrument model. The resultant conditional alphas may be easily estimated and are consistent with conditional efficient set theory. The resultant CIOS alphas differ substantially from both the unconditional alpha and other regression based conditional alphas in the extant literature.

The following theorem proves that the vector of CIOS alphas, $\alpha_{2|z}$, and CIOS betas, $\beta_{2|z}$, can be obtained directly from a simple multivariate regression in unconditional returns and instruments' values.

THEOREM: The conditional alpha vector, given instrument values, Z, is equal to $\alpha_{2|z} = \delta_2 + \lambda_2 Z$, where δ_2 is the intercept and λ_2 is the coefficient matrix of the instruments in the unconditional regression, given by

$$R_2 = \delta_2 + \phi_2 R_1 + \lambda_2 Z + \xi_2, \tag{7}$$

where ϕ_2 is the coefficient matrix of the spanning portfolios.

The proof is provided in Appendix 1.

Therefore, the conditional alpha may be obtained simply by running the unconditional regression (7) to obtain the coefficients, δ_2 and λ_2. For a particular realization of the instrument vector, Z, the sum $\delta_2 + \lambda_2 Z$ is equivalent to the true conditional alpha in (6). Equation (7) demonstrates that given the conditional linearity of equation (2a) and the conditional homoskedasticity of equation (2b), the conditional alpha is linear in the instruments.[9]

From the proof provided in Appendix 1, it is evident that both the conditional alphas and conditional betas given by (3a) and (3b) may be obtained from the unconditional regression specified by equation (7).[10] However, the proper regression form used to determine the conditional alphas is dependent on how instruments are related to returns. Assuming a linear model for the effect of instruments on returns results in our simple method for estimating conditional alphas and conditional betas.

An important question is whether the extant literature's regressions, compared with our regressions, produce alphas that are parametrically equal to the true CIOS conditional alphas defined by (6).

4.1 Other Literature's Regressions and Conditional Alphas

Related seminal works include Campbell (1987) and Shanken (1990) who posit

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[9] We thank Raymond Kan for this insight.

[10] An alternative estimation approach to the regression (7) is to transform the return variables, R_2 and R_1, by employing the respective eigenvalues and corresponding eigenvectors of their covariance matrix. This approach is similar to that used in principal component analysis where the eigenvectors are orthogonal. We thank Dave Jobson for this suggestion.

models in which first and second moments are linear in the instruments.[11] Ferson and Qian (2004) provide a summary of recent approaches to conditional alpha estimation using coefficients that are linearly related to underlying information instruments.[12] For example, the regressions from Ferson and Schadt (FS, 1996) and Christopherson, Ferson, and Glassman (CFG, 1998) are respectively,

$$R_{2j,t} = \alpha_{2j}^{FS} + \beta_{2j|z}^{FS} R_{1,t} + \phi_{2j|z}^{FS}\left(R_{1,t} \otimes z_t\right) + \varsigma_{2j,t} \qquad (8a)$$

and

$$R_{2j,t} = \varphi_{2j|z}^{CFG} + \theta_{2j|z}^{CFG} z_t + \beta_{2j|z}^{CFG} R_{1,t} + \phi_{2j|z}^{CFG}\left(R_{1,t} \otimes z_t\right) + \zeta_{2j,t}\,, \qquad (8b)$$

where $z_t = Z_t - \mu_Z$, μ_z is the vector of mean instrument values, $\otimes$ is the Kronecker product, and time subscripts indicate that the regressions are estimated from inter-temporal data. Subsequent to estimating the coefficients, substituting a value of the information vector, Z_t, determines the period to which the conditional alpha applies. The CFG conditional alpha may be written as, $\alpha_{2j|z}^{CFG} = \varphi_{2j|z}^{CFG} + \theta_{2j|z}^{CFG} z_t$.

To see how a model of time varying coefficients may produce these conditional alpha measures, consider an unconditional regression equation,

$$R_{2t} = c_0 + c_{1,t} R_{1t} + \nu_t$$

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[11] Shanken (1990) considers a model in which coefficients are assumed to be linear in the instruments to examine a zero intercept restriction. We favor a direct approach to determine conditional regression parameters, where the dependence on the instruments arises from the conditional moment specifications thereby maintaining internal consistency.

[12] Other examples of this approach may be found in Ferson and Schadt (1996), Ferson and Warther (1996), Christopherson, Ferson, and Glassman (1998), Becker, Ferson, Myers, and Schill (1999), Zheng (1999), and Christopherson, Ferson, and Turner (1999).

with a time varying slope coefficient, $c_{1,t} = \frac{Cov\left(R_{2t}, R_{1t} \mid Z_t\right)}{Var\left(R_{1t} \mid Z_t\right)} = c_{10} + c_{11} z_t$. Substituting the time varying coefficient for the slope coefficient yields

$$R_{2t} = c_0 + c_{10} R_{1t} + c_{11} R_{1t} \otimes z_t + \nu_t$$

which is of the general form given by FS in (8a). Relaxing the intercept and assuming linearity in the instruments, yields the CFG regression (8b) with $c_{0,t} = c_{00} + c_{01} z_t$. Our preference is to develop the regression parameter functional form directly from the relation between the conditional return moments and the theory of the efficient set; rather than adopting a specific ad hoc functional form in the spanning regression setup.

Comparisons of (7) with (8a) and (8b) show that (7) does not include terms involving the product of instruments and spanning portfolio returns, $R_{1,t} \otimes z_t$, as is the case in FS and CFG. Equation (8b) is similar to (7) in that it does measure changes in the intercept related to the instruments. The FS conditional alpha effectively evaluates equation (8a) at the mean of the spanning portfolios and at the mean of the Kronecker product of the spanning portfolios and the instruments. Unfortunately, the specification of the 'conditional intercept' in (8a) is constant and does not vary over time with changes in realized instrument values. Comparing equations (7) and (8a), the FS conditional alpha is essentially obtained from a regression excluding the instruments, *Z*, but including a redundant regressor, $R_{1,t} \otimes z_t$. This has implications for the unbiasedness and efficiency of FS alpha estimates.

The CFG regression captures the effects of time-varying alphas with $\varphi_{2j|z}^{CFG} + \theta_{2j|z}^{CFG} z_t$ and time-varying betas within the terms $\beta_{2j|z}^{CFG} R_{1,t} + \phi_{2j|z}^{CFG}\left(R_{1,t} \otimes z_t\right)$. Regressions (8a) and (8b) may capture some interesting time variation in the coefficients of an unconditional regression; however, the derived alphas, conditional on instrument values and a model of conditional return moments (such as (2a) and (2b)), are not in general equivalent to the conditional alpha implied by the CIOS and given in equation (3a); i.e.,

$$\varphi_{2j|z}^{CFG} + \theta_{2j|z}^{CFG} z_t \neq \mu_{2j|z} - \Sigma_{2_j 1|z} \Sigma_{11|z}^{-1} \mu_{1|z} .$$

The implication is that the conditional alphas from these regressions do not have the same economic meaning as the conditional alpha based on the IOS conditional return moments.[13] The CFG conditional alphas will be empirically similar to our conditional alphas when the final regressor has little impact on the estimated parameters. Nonetheless, the efficiency of the CFG alpha will likely be poor given the inclusion of this redundant regressor. It is difficult to hypothesize a set of restrictions for conditional moments in which the Ferson and Schadt (1996) conditional alpha will be similar to our CIOS alpha.

In the next section, we examine a twenty-year sample of monthly returns on 341 mutual fund returns to determine the empirical importance of our findings for a given instrument set. We also provide finite sample properties of our proposed measure and the literature's alpha estimates from a bootstrap analysis. We present simulation results in the Section 6 to clarify the inefficiencies that we expect to see in the estimated coefficients in the FS and the CFG approaches.

[13] Ferson and Siegel (2006), when testing portfolio efficiency in the presence of conditioning information using lagged instruments, find that the multiplicative approach is inconsistent with mean variance efficiency. The authors use a framework based on unconditional mean variance efficiency (following Hansen and Richard 1987) with respect to conditioning information where the portfolio weights may be any well-behaved functions of the conditioning information.

5. Empirical Analysis

Our empirical analysis assumes that a single index portfolio and the risk-free asset span the investment opportunities of passive portfolios; but not the investment opportunities of a large number of actively managed mutual funds. Thus, all actively managed funds will have nonzero alphas in general.

5.1 The Data and Unconditional Parameters

We choose an investible sample of domestic equity fund returns from the Morningstar Principia database. We impose a number of data screens to consider only domestic equity funds with an accompanying investment style that has been categorized into one of nine value and size types as classified by Morningstar. As a further check on fund composition, we restrict our analysis to funds with 100 percent of their investments within North America measured at the close of November 2005, December 2005, January 2006, or February 2006. Finally to focus our analysis on funds that are investible, we exclude funds that are closed to new investors and funds that require an initial minimum purchase amount greater than $25,000.[14] Measured total returns are net of management, administrative, and 12b-1 fees but do not include any brokerage costs or loads. Because many funds offer multiple classes on the same underlying fund, we choose only the largest mutual fund class (as measured by net assets) when there are multiple class offerings.[15] After eliminating redundant share class offerings and only considering funds with available data over the full sample period, available data exists for 341 mutual

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[14] Our sample screens are similar to Del Guercio and Tkac (2002) who consider the flow of funds to mutual funds and pension funds. Morningstar makes every effort to ensure accuracy of this data, but cannot guarantee completeness and accuracy.

[15] In a similar context, Gaspar, Massa, and Matos (2006) also choose to retain the largest mutual fund class when collecting mutual fund data from the CRSP US Mutual Fund database.

funds.[16] We form five equally weighted style portfolios defined into categories that include growth, value, blend (a combination of growth and value stocks), specialty (funds focusing on industry sectors such as natural resources, health etc.) and other, a classification similar to Ferson and Schadt (1996).

We consider a five instrument information set that is comparable to the instruments considered in Ferson and Schadt (1996) and includes the lagged one-month Treasury bill return from Ibbotson and Associates (percent per month), the lagged S&P500 percentage dividend yield from CRSP for the previous twelve months, the lagged slope of the term structure specified as the difference in annual percentage yields on ten-year Treasury bonds and three-month Treasury bills, the lagged credit quality yield spread measured by the difference in annual percentage yields on Moody's seasoned BAA-rated corporate bonds and AAA-rated corporate bonds both from the St. Louis Federal Reserve, and an indicator variable for the month of January. These variables proxy for the information set impacting conditional asset excess return moments.[17] After combining the instrument sets and the mutual fund returns, available data exists from April 1986 through November

[16] Our sample may be affected by survivorship bias issues given our desire for a lengthy time series of data on available funds. Excellent reviews of this issue as it relates to mutual fund performance may be found in Grinblatt and Titman (1989, 1992), Brown and Goetzmann (1995), Malkiel (1995), Carhart (1997), Elton, Gruber, and Blake (1996, 2001) and Carhart, Carpenter, Lynch, and Musto (2002). Carhart et al. (2002) demonstrate that the bias in estimates of average annual performance increases in sample length but at a declining rate. Our bootstrap analysis is comparable to Kosowski, Timmerman, Wermers, and White (2006) who find comparable bootstrapped alpha estimates for data lengths ranging from 18 to 120 months of observations. These authors conclude that survivorship bias has little impact on their bootstrap results.

[17] We calculate the full covariance matrix of fund excess returns, index excess returns, and the instruments, $\begin{bmatrix} \Sigma_{zz} & \Sigma_{z1} & \Sigma_{z2} \\ \Sigma_{1z} & \Sigma_{11} & \Sigma_{12} \\ \Sigma_{2z} & \Sigma_{21} & \Sigma_{22} \end{bmatrix}$. These values facilitate empirical verification of the analytic results presented throughout.

2005. Of the mutual funds considered, 341 have full returns data for the entire sample period.

The second and third columns in Table 1, Panel A report the unconditional sample means and standard deviations for excess returns for the style portfolios. During the 236 month period from April, 1986 to November, 2005, the sample means of the style portfolios excess returns range from 0.38 to 0.81 percent per month. Similarly, sample standard deviations of our style portfolios range from 2.80 to 6.35 percent per month. The sample average S&P500 index excess return during this period is approximately 0.43 percent per month or approximately 5.3 percent on an annualized basis.

5.2 Comparison of Conditional Alphas

Next, we compare the statistical properties of the CIOS, FS and CFG alpha estimates. In Table 1, we report the unconditional alpha, unconditional beta, and conditional alpha for five potential instrument value realizations, Z_{1+} through Z_{5+} for the five style portfolios. We introduce five potential instrument realizations to provide some intuition regarding how conditional alphas vary with economic state variables. The unconditional alpha estimates provide results computed at the instrument means. Our final five columns of conditional alphas are evaluated at the conditional mean of four of the five instruments. The fifth instrument is then increased by one standard deviation to provide intuition regarding how the conditional alpha changes with that instrument. For example, the column labeled Z_{3+} evaluates the conditional alpha at the mean of instrument 1, 2, 4 and 5, and at one standard deviation above the mean for the third instrument (the lagged term structure slope variable). The reported p-values for the unconditional alpha and beta are from the simple regression with no instruments included. For the conditional alpha estimates, we report the theoretical p-values for the test that the conditional alpha is equal to zero for each instrument realization assuming multivariate normality. These values are shown in Panel A below the parameter estimates.

*** Insert Table 1 about here ***

To consider the importance of potential violations of normality and homoskedasticity, we also provide bootstrapped 95 percent confidence intervals under the null hypothesis for all conditional alphas for a variety of instrument realizations. The bootstrapped confidence interval evaluated at the instrument means is reported under the unconditional alpha estimates. The bootstrapping procedure is described in Appendix 2.

From Panel A, we observe that the unconditional alphas are significant at the one percent level for all the portfolios except the growth portfolio. The bootstrapped 95 percent confidence intervals for the conditional alphas at the instrument means lie outside the interval for all portfolios except for the growth portfolio. We also observe that all unconditional beta estimates are significantly different than zero. The final five columns report the reported conditional alpha estimates computed at Z_{1+} through Z_{5+}. The first three columns show only one case of significance when these instruments are increased one standard deviation from their mean. The final two columns shows that the conditional alphas are highly significant when the lagged credit quality spread variable or the January dummy variable is larger than its mean by one standard deviation. For example, when the lagged credit spread variable is increased, the value portfolio conditional alpha is 0.27. The related conditional alphas for the blend and other portfolio also increase relative to their values at the mean of the instrument set. We note that in every reported instance, the bootstrapped 95 percent confidence interval provides the same inference as the theoretical p-values reported.

Panels B and C report the related conditional alphas for the Ferson and Schadt (FS) and Christopherson, Ferson, and Glassman (CFG) methods. By construction, the reported FS alphas or the 95 percent bootstrapped confidence intervals in Panel B do not vary across the Z_{1+} through Z_{5+} columns. Panel C shows that the reported CFG point estimates are similar to the CIOS conditional alphas in magnitudes and significance levels for various instrument realizations. The inferences provided from the 95 percent confidence intervals align well with those in Panel A. In contrast, the reported CFG estimates are often economically quite different than the reported CIOS estimates. As an example, the CFG alpha for the specialty fund evaluated at Z_{5+} is 0.15 or 26 percent larger than the comparable CIOS alpha from Panel A. In addition, the reported 95 percent bootstrapped confidence intervals for the CFG estimates are wider than the comparable Panel A confidence intervals. This inefficiency is not surprising given that the CFG alphas are developed from a regression with a redundant regressor. In the next section, we present a simulation to examine the relative efficiency of the various estimators.

5.3 Comparison of the Efficiency and Bias of the Various Estimators under the Alternative

In this section, we report bootstrapping results to compare the various conditional alpha estimators with respect to both bias and efficiency under the alternative of positive conditional performance. For each of the CIOS, Ferson and Schadt (FS, 1996), and Christopherson, Ferson, and Glassman (CFG, 1998) conditional alpha estimators, we follow a procedure similar to that described in Appendix 2.

Abnormal performance is introduced into the generated regressors using $\hat{R}_{2j,t} = abnorm_t + \hat{\phi}_{2j} R_{1,t} + \hat{\xi}_{2j,t}$ where abnormal performance is defined as,

$$abnorm_t = \frac{0.3}{6} + \frac{0.3}{6\bar{Z}_1} Z_{1,t} + \frac{0.3}{6\bar{Z}_2} Z_{2,t} + \frac{0.3}{6\bar{Z}_3} Z_{3,t} + \frac{0.3}{6\bar{Z}_4} Z_{4,t} + \frac{0.3}{6\bar{Z}_5} Z_{5,t} .$$

Taking unconditional expectations of this equation shows that the unconditional expected abnormal performance is 0.3 and that this abnormal performance is allocated equally (on average) to each of the five instruments in the analysis.

For each of 5,000 bootstrapping simulations, we calculate the root mean squared error (*rmse*), $\sqrt{\frac{\sum_{t=1}^{T} \left(\alpha_{true} - \hat{\alpha}\right)^2}{T}}$, percentage root mean squared error (*% rmse*), $\sqrt{\frac{\sum_{t=1}^{T} \frac{\left(\alpha_{true} - \hat{\alpha}\right)^2}{\alpha_{true}}}{T}}$, bias, $\frac{\sum_{t=1}^{T} \left(\alpha_{true} - \hat{\alpha}\right)}{T}$, and percentage bias, $\frac{\sum_{t=1}^{T} \frac{\left(\alpha_{true} - \hat{\alpha}\right)}{\alpha_{true}}}{T}$, where α_{true} is the true conditional alpha given by $abnorm_t$, $\hat{\alpha}$ is the estimated conditional alpha (CIOS, FS or CFG), and T = 236. Table 2 reports the median value of these quantities for each conditional alpha estimate and each style portfolio.

*** Insert Table 2 about here ***

The final two columns of Table 2 demonstrate a substantial empirical bias in the FS conditional alpha. For instance, for most style portfolios the bias in the FS conditional alpha is typically a full magnitude larger than the comparable biases for either the CIOS or CFG estimators. The CIOS and CFG alphas have comparably small percentage biases for these instruments and style portfolios. These results are not surprising since the FS approach has an important missing regressor in its specification.

The median values for the *rmse* and *% rmse* are reported in the third and fourth columns. The CIOS values appear similar, but consistently smaller than the comparable CFG medians. For example, the median *rmse* for the blend portfolio is 0.0176 for the CIOS alpha and 0.0184 for the CFG conditional alpha. This result is not unexpected given the redundant CFG regressor. Interestingly, the FS estimator performs quite well relative to the CIOS and CFG estimators in terms of *rmse*. For example, for the growth and specialty portfolios, the FS alpha shows a markedly smaller median *rmse* and median *% rmse*. It is possible that the five instruments commonly employed in this literature give rise to a large amount of estimation error in the linear conditional alpha forecast, resulting in large *rmse* values. We examine this issue more closely in a controlled single instrument simulation in the next section.

6. Extensions

6.1 Relaxing Distributional Assumptions

Our general specification in equation (3a) need not rely on a linear model of returns and instrument values. We adopt these specifications as a natural beginning point given their common usage in specifying conditional moment behavior. In general, equation (3) may be implemented in virtually any model for conditional means (linear or nonlinear) and conditional covariances (using either homoskedastic or heteroskedastic model disturbances). For example, conditional means may be derived from a nonlinear forecast model and conditional covariances may be specified by a multivariate mixed GARCH model. What is important is that the conditional investment opportunity set and the resultant performance measures are dependent upon these moment specifications.

To examine the sensitivity of our results to sample size and error distribution assumptions, we consider a simplified simulation with varying sample sizes from a normal or Student *t*-distribution. We generate data under the null hypothesis using the following model, $R_{2t} = a + bR_{1t} + cZ_t + \xi_{2t}$ with parameter values of $a = 0.15$, $b = 0.8$, and $c = 0.05$. The regressors, R_{1t} and Z_t, and the model disturbance, ξ_{2t}, are generated as $R_{1t} = 0.43 + 4.41rv_{1t}$, $Z_t = 1.72 + 1.13rv_{2t}$, and $\xi_{2t} = 0.5rv_{3t}$ where rv_{1t}, rv_{2t}, and rv_{3t} are random variables drawn from either a Student *t* or normal distribution. We assume rv_{3t} is uncorrelated with rv_{1t} and rv_{2t}. Further, we consider correlations between R_{1t} and Z_t of 0, 0.5, and 0.9 for sample sizes of T=100, 200, and 1,000. The true conditional alpha is defined as $\alpha_{truet} = a + cZ_t$.

We then estimate equation (7), (8a), or (8b) for the CIOS, FS, or CFG models, respectively, and calculate the model specific conditional alpha, $\alpha_{est,t}$. We calculate the *% rmse* for each approach. The above process is replicated for N=5,000 simulations.

To compare the relative efficiency of the CIOS approach versus the FS approach, we calculate the percentage efficiency gain across replications as, $\frac{1}{N}\sum_{t=1}^{N}\frac{\left(FS_{\%rmse} - CIOS_{\%rmse}\right)}{CIOS_{\%rmse}}$. The related measure for the CFG estimator is constructed in an equivalent manner. Table 3 reports the mean and median value for the percentage efficiency gain comparing the CIOS to either the FS or CFG approaches. An entry of 0.10 in the table implies that the CIOS estimator is 10 percent more efficient than the alternative estimator at the mean or median across all simulations. Means are reported first followed by medians in parentheses.

*** Insert Table 3 about here ***

Table 3 demonstrates that the CIOS approach is more efficient than both CFG and FS approaches for a variety of sample sizes, correlations between regressors, and sample sizes. The CIOS approach strongly dominates the FS approach for virtually any simulation setup considered. Similarly, we observe a marked improvement in the CIOS approach over the CFG approach for all reported means of $\%\,rmse$ values. Median $\%\,rmse$ values appear marginally better for the CIOS approach in all situations as well; however, the results are similar. These results suggest that the CFG approach gives rise to some important outliers in the $\%\,rmse$ values. It is likely that the marginal improvements of the CIOS estimator versus the CFG estimator in Table 2 are reflective of the medians in Table 3.

Overall, the simulation results do not indicate that the finite sample properties of the CIOS conditional alpha estimates are weakened under alternative error distributions, regressor correlations, or sample sizes. The results suggest our conceptual and empirical findings will be robust to a variety of alternative modeling assumptions.[18]

6.2 Other Conditional Performance Measures

One of the important benefits of our development is that the resultant conditional alpha retains all of the intuition regarding efficient set mathematics when moments are stated in conditional form. In particular, the intuition regarding Sharpe ratios and

[18] An interesting extension that we defer to future work is to extend our specification to consider nonlinear conditional means and an explicit model of conditional heteroskedasticity such as a multivariate GARCH specification.

Appraisal ratios is retained in terms of conditional portfolio management. In this section, we provide a link to the extant efficient set literature.

The maximum increase in the Sharpe ratio, obtained by combining the N_{p1} passive portfolios and an active mutual fund versus holding only the passive tangency portfolio, is a function of the CIOS alpha .[19] An investor's maximum reward to volatility ratio is obtained by combining the passive optimum portfolio with an actively managed mutual fund in optimal proportions. The resultant fund position is a long or a short position depending on good or bad performance, respectively.

The squared Sharpe ratio of the new optimal portfolio formed from the passive optimum portfolio and the active fund, *j*, is

$$Sh^2_{Opt|z} = \frac{\alpha^2_{2_j|z}}{\sigma^2_{e_{2_j}|z}} + Sh^2_{1|z}, \text{ for } j = 1,2,...N_{p2}, \tag{9}$$

where $Sh^2_{1|z}$ is the squared Sharpe ratio for the passive portfolio, $\alpha^2_{2_j|z}$ is the conditional alpha for the active fund, and $\sigma^2_{e_{2_j}|z}$ is the conditional residual variance in the active fund after removing the effect of both the spanning assets and the instruments. The conditional Sharpe ratio of the formerly optimal passive portfolio is,

$$Sh_{1|z} = \frac{\mu_{1|z}}{\sigma_{1|z}}$$

[19] See Treynor and Black (1973) and Jobson and Korkie (1984) for a description.

and the conditional appraisal ratio of fund *j* is

$$\frac{\alpha_{2_j|z}}{\sigma_{e_{2_j}|z}}. \tag{10}$$

Notice that the sign of alpha is hidden in the increased Sharpe performance expression due to the square. It can be shown, however, that any nonzero alpha contributes to improved performance and the alpha sign corresponds to the sign of the optimal position of the fund, *j*, in the optimally combined portfolio of the N_{p1} passive spanning portfolios and the fund. Therefore, large positive alphas are indicative of a positively performing fund, ceteris paribus. In addition, alpha ranking of funds is unreliable if funds differ in their unsystematic risk, $\sigma^2_{e_{2_j}|z}$. This is because the appraisal ratio and Sharpe ratio increase are a function of both alpha and the unsystematic risk.

7. Concluding Remarks

We develop a parametric conditional alpha performance model that is derived directly from the specification for conditional asset returns. We develop a conditional investment opportunity (CIOS) set alpha and provide an analytic link between the conditional CIOS alpha and the unconditional Jensen's alpha. The developed conditional alpha differs from other popular conditional alphas developed in the literature.

Our proposed conditional CIOS alpha may be constructed directly from the conditional investment opportunity set's excess return moments and instrument moments. Following conventional methods, we show how the conditional CIOS alpha may also be obtained in a simple unconditional regression that includes the information instruments.

Our analytic differences with existing conditional alphas show parametric differences with existing measures, absent estimation issues.

Using regression analysis, bootstrap replications, and simulation analysis, we demonstrate that the conditional alphas from the extant literature are not equivalent to the conditional alphas implied by the conditional investment opportunity sets. Our approach is interesting from an empirical perspective because it can be generalized to hold under conditions such as a nonlinear specification for conditional returns or conditional heteroskedasticity of the factor model residuals. Future research is warranted to determine a parsimonious model for conditional expected returns and conditional volatilities with application to conditional performance measures.

Appendix 1: Proof of the Theorem

<u>Proof</u>:

The multivariate regression (7) may be rewritten as,

$$R_2 = \delta_2 + \begin{bmatrix} \phi_2 & \lambda_2 \end{bmatrix} \begin{bmatrix} R_1 \\ Z \end{bmatrix} + \xi_2 ,$$

with the slope coefficient vector given by

$$\begin{bmatrix} \phi_2 & \lambda_2 \end{bmatrix} = \begin{bmatrix} \Sigma_{21} & \Sigma_{2z} \end{bmatrix} \begin{bmatrix} \Sigma_{11} & \Sigma_{1z} \\ \Sigma_{z1} & \Sigma_{zz} \end{bmatrix}^{-1} . \qquad \text{(A1.1)}$$

Using an expression for the partitioned form of the inverse (cf., Judge et. al, 1985), we have

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{1z} \\ \Sigma_{z1} & \Sigma_{zz} \end{bmatrix}^{-1} = \begin{bmatrix} \left(\Sigma_{11} - \Sigma_{1z}\Sigma_{zz}^{-1}\Sigma_{z1} \right)^{-1} & -\left(\Sigma_{11} - \Sigma_{1z}\Sigma_{zz}^{-1}\Sigma_{z1} \right)^{-1} \Sigma_{1z}\Sigma_{zz}^{-1} \\ -\Sigma_{zz}^{-1}\Sigma_{z1}\left(\Sigma_{11} - \Sigma_{1z}\Sigma_{zz}^{-1}\Sigma_{z1} \right)^{-1} & \Sigma_{zz}^{-1} + \Sigma_{zz}^{-1}\Sigma_{z1}\left(\Sigma_{11} - \Sigma_{1z}\Sigma_{zz}^{-1}\Sigma_{z1} \right)^{-1} \Sigma_{1z}\Sigma_{zz}^{-1} \end{bmatrix}$$

assuming Σ_{zz} and $\left(\Sigma_{11} - \Sigma_{1z}\Sigma_{zz}^{-1}\Sigma_{z1} \right)$ are nonsingular.

Substituting this expression into (A1.1) results in the partitioned coefficient matrix

$$\begin{aligned}
\begin{bmatrix} \phi_2 & \lambda_2 \end{bmatrix} &= \begin{bmatrix} \Sigma_{21} & \Sigma_{2z} \end{bmatrix} \begin{bmatrix} \Sigma_{11} & \Sigma_{1z} \\ \Sigma_{z1} & \Sigma_{zz} \end{bmatrix}^{-1} \\
&= \begin{bmatrix} \Sigma_{21}\Sigma_{11|z}^{-1} - \Sigma_{2z}\Sigma_{zz}^{-1}\Sigma_{z1}\Sigma_{11|z}^{-1} & -\Sigma_{21}\Sigma_{11|z}^{-1}\Sigma_{1z}\Sigma_{zz}^{-1} + \Sigma_{2z}\left(\Sigma_{zz}^{-1} + \Sigma_{zz}^{-1}\Sigma_{z1}\Sigma_{11|z}^{-1}\Sigma_{1z}\Sigma_{zz}^{-1} \right) \end{bmatrix} \\
&= \begin{bmatrix} \Sigma_{21|z}\Sigma_{11|z}^{-1} & -\Sigma_{21}\Sigma_{11|z}^{-1}\Sigma_{1z}\Sigma_{zz}^{-1} + \Sigma_{2z}\left(\Sigma_{zz}^{-1} + \Sigma_{zz}^{-1}\Sigma_{z1}\Sigma_{11|z}^{-1}\Sigma_{1z}\Sigma_{zz}^{-1} \right) \end{bmatrix} \\
&= \begin{bmatrix} \Sigma_{21|z}\Sigma_{11|z}^{-1} & \Sigma_{2z}\Sigma_{zz}^{-1} - \Sigma_{21|z}\Sigma_{11|z}^{-1}\Sigma_{1z}\Sigma_{zz}^{-1} \end{bmatrix} = \begin{bmatrix} \Sigma_{21|z}\Sigma_{11|z}^{-1} & b_2 - \beta_{2|z}b_1 \end{bmatrix} \\
&= \begin{bmatrix} \Sigma_{21|z}\Sigma_{11|z}^{-1} & \Sigma_{2z|1}\Sigma_{zz|1}^{-1} \end{bmatrix} .
\end{aligned}$$

(A1.2)

From (A1.2), we observe that the conditional beta is identical to the regression coefficient matrix, ϕ_2,

$$\beta_{2|z} = \phi_2 = \Sigma_{21|z}\Sigma_{11|z}^{-1},$$

and

$$\lambda_2 = b_2 - \beta_{2|z}b_1 = \Sigma_{2z|1}\Sigma_{zz|1}^{-1}.$$

The regression constant may now be obtained after substitution for ϕ_2 and λ_2 as

$$\begin{aligned}\delta_2 &= \mu_2 - \phi_2\mu_1 - \lambda_2\mu_z = \mu_2 - \beta_{2|z}\mu_1 - \left(b_2 - \beta_{2|z}b_1\right)\mu_z \\ &= \left(\mu_2 - b_2\mu_z\right) - \beta_{2|z}\left(\mu_1 - b_1\mu_z\right).\end{aligned}$$

The sum of the first and third terms from equation (7) is

$$\begin{aligned}\delta_2 + \lambda_2 Z &= \left(\mu_2 - b_2\mu_z\right) - \beta_{2|z}\left(\mu_1 - b_1\mu_z\right) + \left(b_2 - \beta_{2|z}b_1\right)\left(\mu_z + z\right) \\ &= \left(\mu_2 + b_2 z\right) - \beta_{2|z}\left(\mu_1 + b_1 z\right)\end{aligned}. \qquad \text{(A1.3)}$$

Comparison of (A1.3) with (3a) demonstrates that the conditional alpha is

$$\alpha_{2|z} = \delta_2 + \lambda_2 Z$$

as required. QED.

Appendix 2: The Bootstrap Procedure

Our bootstrap procedure is similar to Kosowski et al. (2006). We first use the CIOS specification given by equation (7) to compute ordinary least squares (OLS) coefficient estimates and to generate residuals, $\hat{\xi}_{2j,t}$ for $t = 1,2,...,T$ **,** using monthly excess returns for each fund $j = 1,2,...,N$ **,**

$$R_{2j,t} = \delta_{2j} + \phi_{2j} R_{1,t} + \lambda_{2j} Z_t + \xi_{2j,t} \,, \qquad \text{(A2.1)}$$

in the sample. For each bootstrap iteration, we draw a pseudo time series of T resampled residuals and construct a time-series of bootstrapped returns $\hat{R}_{2j,t} = \hat{\phi}_{2j} R_{1,t} + \hat{\xi}_{2j,t}$ for fund *j* assuming zero abnormal performance. We then estimate the regression

$$\hat{R}_{2j,t} = \delta_{2jb} + \phi_{2jb} R_{1,t} + \lambda_{2jb} Z_t + \xi_{2jb,t} \qquad \text{(A2.2)}$$

using the bootstrapped returns, for $b = 1,2,...,5000$. This provides an empirical distribution of the conditional alphas under the null hypothesis.

The underlying conditional alpha will change with the underlying instrument realization. To provide a succinct summary of our results, we consider six cases for the realized instrument set. First, we consider all instruments at their mean realizations. Next we hold four of the five instruments at their mean values and let the fifth instrument be given by its mean plus one standard deviation. This process is used to define Z_{1+} through Z_{5+}. For example, the column labeled Z_{3+} evaluates the conditional alpha at the mean of instrument 1, 2, 4 and 5, and one standard deviation above the mean for the third instrument.

We compare the estimated CIOS alpha, $\hat{\delta}_{2j} + \hat{\lambda}_{2j} Z_t$, from equation (A2.1) to the distribution of coefficients from the regression in equation (A2.2), $\hat{\delta}_{2jb} + \hat{\lambda}_{2jb} Z_t$, for $b = 1,2,...,5000$. The instrument is set to its mean or one of Z_{1+} through Z_{5+} in both (A2.1) and (A2.2). We repeat the procedure for the FS and CFG alphas by replacing (A2.2) with the appropriate FS and CFG regression models.

References

Ahn, Dong-Hyun, H. Henry Cao, and Stephane Chrétien, 2003, Portfolio performance measurement: A no arbitrage bounds approach, unpublished, *University of North Carolina, Chapel Hill.*

Barras, Laurent, O. Scaillet, and Russ Wermers, 2006, False discoveries in mutual fund performance: Measuring luck in estimated alphas, FEN Working paper.

Becker, Connie, Wayne E. Ferson, David Myers, and Michael Schill, 1999, Conditional market timing with benchmark investors, *Journal of Financial Economics*, 52, 119-148.

Bollerslev, Tim, 1986, Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics* 31, 307-327.

Brown, Stephen and William Goetzmann, 1995, Performance persistence, *Journal of Finance*, 50, 679-698.

Campbell, John Y., 1987, Stock returns and the term structure, *Journal of Financial Economics*, 18, 373-399.

Carhart, Mark M., 1997, On the persistence of mutual fund performance, *Journal of Finance*, 52, 57-82.

Carhart, Mark M., Jennifer N. Carpenter, Anthony W. Lynch, and David K. Musto, 1997, On the persistence of mutual fund performance, *Journal of Finance*, 52, 57-82.

Cochrane, John H. and Jésus Saá-Requejo, 2000, Beyond arbitrage: "Good deal" asset price bounds in incomplete markets, *Journal of Political Economy*, 106, 79-119.

Chen, Zhiwu and Peter J. Knez, 1996, Portfolio performance measurement: Theory and applications, *Review of Financial Studies*, 9, 511-555.

Christopherson, Jon A., Wayne E. Ferson, and Debra A. Glassman, 1998, Conditioning manager alpha on economic information: Another look at the persistence of performance, *Review of Financial Studies*, 3, 32-43.

Christopherson, Jon A., Wayne E. Ferson, and Andrew L. Turner, 1999, Performance evaluation using conditional alphas and betas, *Journal of Portfolio Management*, 26, 59-72.

Del Guercio, Diane, and Paula A. Tkac, 2002, The determinants of the flow of funds of managed portfolios: Mutual funds vs. pension funds, *Journal of Financial and Quantitative Analysis*, 37, 4, 523-557.

Elton, Edwin J., Martin J. Gruber, and Christopher R. Blake, 1996, Survivorship bias and mutual fund performance, *Review of Financial Studies*, 9, 1097-1120.

Elton, Edwin J., Martin J. Gruber, and Christopher R. Blake, 2001, A first look at the accuracy of the CRSP mutual fund database and a comparison of the CRSP and Morningstar mutual fund databases, *Journal of Finance*, 56, 2415-2430.

Fama, Eugene F., and Kenneth R. French, 1995, Size and book-to-market factors in earnings and returns, *Journal of Finance*, 50, 131-155.

Ferson, Wayne E., Steve R. Foerster, and Donald Keim, 1993, General tests of latent variable models and mean variance spanning, *Journal of Finance*, 48, 131-156.

Ferson, Wayne E. and Rudi Schadt, 1996, Measuring fund strategy and performance in changing economic conditions, *Journal of Finance*, 51, 425-462.

Ferson, Wayne E. and Vincent A. Warther, 1996, Evaluating fund performance in a dynamic market, *Financial Analysts Journal*, 52, 20-28.

Ferson, Wayne E. and Meijun Qian, 2004, *Conditional Performance Evaluation*, Research Foundation of CFA Institute, Charlottesville Virginia.

Ferson, Wayne E. and Andrew F. Siegel, 2006, Testing portfolio efficiency with conditioning information, Working Paper, Boston College.

Gaspar, Jose-Miguel, Massimo Massa, and Pdero Matos, 2006, Favoritism in mutual fund families? Evidence on strategic cross-fund subsidization, *Journal of Finance*, 61, 73-104..

Grinblatt, Mark, and Sheridan Titman, 1989, Portfolio performance evaluation: Old issues and new insights, *Review of Financial Studies*, 2, 393-421.

Grinblatt, Mark, and Sheridan Titman, 1992, Performance persistence in mutual funds, *Journal of Finance*, 51, 783-810.

Hansen, Lars P., and Scott F. Richard, 1987, The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models, *Econometrica*, 55, 3, 587-613.

Harvey, Campbell, 1989, Time varying conditional covariances in tests of asset pricing models, *Journal of Financial Economics*, 24, 289-317.

Jensen, Michael, 1968, The performance of mutual funds in the period 1945-1964, *Journal of Finance*, 23, 389-446.

Jobson, Dave J. and Bob Korkie, 1982, Potential performance and tests of portfolio efficiency, *Journal of Financial Economics*, 10, 433-466.

Jobson, Dave J. and Bob Korkie, 1984, A note on the Jensen measure and marginal improvements in portfolio performance, *Journal of Finance*, 39, 245-251.

Judge, George G., William E. Griffiths, R. Carter Hill, Helmut Lutkepohl, and Tsoung-Chao, Lee, 1985, *The theory and practice of econometrics*, second edition, John Wiley and Sons, New York, NY.

Korkie, Bob and Harry J. Turtle, 2002, A mean-variance analysis of self-financing portfolios, *Management Science*, 48, 427-443.

Kosowski, Robert, Allan Timmermann, Russ Wermers, and Hal White, 2006, Can mutual fund "stars" really pick stocks? New evidence from a bootstrap analysis, *Journal of Finance*, 61, 2551-2595.

Lehmann, B., and D. Modest, 1988, The empirical foundations of the arbitrage pricing theory, *Journal of Financial Economics*, 21, 213-254.

Malkiel, Burton G., 1995, Returns from investing in equity mutual funds 1971 to 1991, *Journal of Finance*, 50, 549-572.

MacKinlay, A. Craig, 1987, On multivariate tests of the CAPM, *Journal of Financial Economics*, 18, 341-372.

Roll, Richard, 1977, A critique of the asset pricing theory's tests: Part 1: On past and potential testability of the theory, *Journal of Financial Economics*, 4, 129-176.

Rubinstein, Mark, 2001, Rational markets? Yes or no: The affirmative case, *Financial Analysts Journal*, 57, 15-29.

Shanken, Jay, 1990, Intertemporal asset pricing: An empirical investigation, *Journal of Econometrics*, 45, 1-2, 99-120.

Sharpe, William F., 1966, Mutual Fund Performance, *Journal of Business*, January, 119-138.

Treynor, Jack L. and Fischer Black, 1973, How to use security analysis to improve portfolio selection, *Journal of Business*, 46, 66-86.

Zheng, Lu, 1999, Is money smart? A study of mutual fund investors fund selection ability, *Journal of Finance*, 54, 901-933.

Table 1
Conditional parameter values for various instrument realizations and portfolios

We consider 341 mutual funds over the 236-month sample period from April, 1986 to November, 2005. We form five equal weighted style portfolios, defined into categories blend, growth, specialty, value, and other. We report the sample mean monthly returns and standard deviation of monthly returns for each of these style portfolios. We report the unconditional alpha, unconditional beta, and conditional alpha for the five style portfolios for the 236-month sample period. We consider five information instruments, given by the lagged 1-month T-bill yield, the lagged S&P500 dividend yield, the lagged slope of the term structure specified as the difference in yields of a 10-year Treasury bond and a 3-month Treasury bill, the lagged credit quality spread measured by the difference in yields of Moody's seasoned BAA-rated corporate bonds and AAA-rated corporate bonds, and an indicator variable for the month of January. Results are reported for the potential instrument value realizations, Z_{1+} through Z_{5+} (where Z_{k+} represents a realization of the *k*th instrument at its mean realization plus one standard deviation, and all other instruments are evaluated at their mean). Reported p-values for the unconditional alpha and beta are from the simple regression with no instruments included. Theoretical p-values for the conditional alpha estimates are reported below parameter estimates in Panel A. The bootstrapped 95% confidence intervals are created under the null hypothesis and are reported below the reported p-values in brackets. The 95% confidence interval for the conditional alpha evaluated at the instrument means is reported under the unconditional alpha estimates. Panel B reports results for the Ferson and Schadt (1996) conditional alpha estimates, and Panel C reports results for the Christopherson, Ferson, and Glassman (1998) estimates.

Portfolio	Sample mean return (% per month)	Sample standard deviation (% per month)	Unconditional alpha	Unconditional beta	Conditional alpha estimates for various instrument realizations				
			α_2 (p-value) [95% CI]	β_{21} (p-value)	Z_{1+} (p-value) [95% CI]	Z_{2+} (p-value) [95% CI]	Z_{3+} (p-value) [95% CI]	Z_{4+} (p-value) [95% CI]	Z_{5+} (p-value) [95% CI]
Panel A: CIOS Results									
Value	0.5676	4.0145	0.2255 (0.006) [-0.13,0.13]	0.7961 (0.000)	0.1736 (0.384) [-0.33,0.32]	0.1759 (0.290) [-0.27,0.28]	0.2430 (0.176) [-0.30,0.28]	0.2715 (0.025) [-0.19,0.20]	0.3359 (0.004) [-0.18,0.19]
Blend	0.5477	4.4887	0.1612 (0.006) [-0.09,0.09]	0.8981 (0.000)	0.1395 (0.326) [-0.23,0.23]	0.1565 (0.187) [-0.19,0.19]	0.2038 (0.112) [-0.21,0.21]	0.1967 (0.023) [-0.14,0.14]	0.2772 (0.001) [-0.13,0.13]
Growth	0.5938	5.6675	0.1338 (0.365) [-0.24,0.24]	1.070 (0.000)	0.0197 (0.956) [-0.57,0.59]	0.2367 (0.430) [-0.49, 0.50]	0.1419 (0.661) [-0.50,0.52]	0.1231 (0.571) [-0.35,0.36]	0.4562 (0.031) [-0.33,0.35]
Specialty	0.8096	6.3468	0.3822 (0.002) [-0.20,0.21]	0.995 (0.000)	0.2173 (0.466) [-0.47,0.48]	0.3381 (0.175) [-0.39,0.41]	0.2796 (0.298) [-0.43,0.43]	0.3666 (0.043) [-0.30,0.29]	0.5802 (0.001) [-0.28,0.29]
Other	0.3786	2.7990	0.1444 (0.001) [-0.07,0.07]	0.5451 (0.000)	0.0381 (0.715) [-0.18,0.17]	0.2075 (0.018) [-0.14,0.14]	0.1354 (0.150) [-0.15,0.15]	0.1518 (0.017) [-0.10,0.10]	0.2365 (0.000) [-0.10,0.10]

Panel B: Ferson and Schadt (1996) results						
	α_2 [95% CI]	Z_{1+} [95% CI]	Z_{2+} [95% CI]	Z_{3+} [95% CI]	Z_{4+} [95% CI]	Z_{5+} [95% CI]
Value	0.1902	0.1902	0.1902	0.1902	0.1902	0.1902
	[-0.14,0.13]	[-0.14,0.13]	[-0.14,0.13]	[-0.14,0.13]	[-0.14,0.13]	[-0.14,0.13]
Blend	0.1576	0.1576	0.1576	0.1576	0.1576	0.1576
	[-0.10,0.10]	[-0.10,0.10]	[-0.10,0.10]	[-0.10,0.10]	[-0.10,0.10]	[-0.10,0.10]
Growth	0.1461	0.1461	0.1461	0.1461	0.1461	0.1461
	[-0.24,0.25]	[-0.24,0.25]	[-0.24,0.25]	[-0.24,0.25]	[-0.24,0.25]	[-0.24,0.25]
Specialty	0.4247	0.4247	0.4247	0.4247	0.4247	0.4247
	[-0.20,0.21]	[-0.20,0.21]	[-0.20,0.21]	[-0.20,0.21]	[-0.20,0.21]	[-0.20,0.21]
Other	0.1198	0.1198	0.1198	0.1198	0.1198	0.1198
	[-0.07,0.07]	[-0.07,0.07]	[-0.07,0.07]	[-0.07,0.07]	[-0.07,0.07]	[-0.07,0.07]

Panel C: Christopherson, Ferson, and Glassman (1998) results						
	α_2 [95% CI]	Z_{1+} [95% CI]	Z_{2+} [95% CI]	Z_{3+} [95% CI]	Z_{4+} [95% CI]	Z_{5+} [95% CI]
Value	0.1986	0.2135	0.1215	0.2735	0.2300	0.3988
	[-0.14,0.13]	[-0.34,0.33]	[-0.27,0.28]	[-0.30,0.29]	[-0.20,0.20]	[-0.20,0.20]
Blend	0.1643	0.1574	0.1588	0.2111	0.2091	0.3038
	[-0.10,0.10]	[-0.24,0.23]	[-0.19,0.20]	[-0.21,0.21]	[-0.14,0.14]	[-0.15,0.15]
Growth	0.1587	0.0008	0.2775	0.1220	0.1627	0.4105
	[-0.24,0.25]	[-0.58,0.60]	[-0.49,0.50]	[-0.52,0.54]	[-0.36,0.37]	[-0.36,0.37]
Specialty	0.4363	0.3181	0.3718	0.3683	0.4061	0.7285
	[-0.20,0.21]	[-0.48,0.49]	[-0.39,0.42]	[-0.44,0.44]	[-0.31,0.30]	[-0.31,0.32]
Other	0.1259	0.0382	0.1805	0.1355	0.1279	0.2513
	[-0.07,0.07]	[-0.18,0.17]	[-0.14,0.14]	[-0.16,0.16]	[-0.10,0.11]	[-0.11,0.11]

Table 2
A comparison of efficiency gains across different estimation approaches under the alternative hypothesis

We compute the CIOS, Ferson and Schadt (FS 1996), and the Christopherson, Ferson, and Glassman (CFG 1998) conditional alphas for five style portfolios defined as blend, growth, specialty, value, and other for the 236-month sample period from April, 1986 to November, 2005. The five information instruments are given by the lagged 1-month T-bill yield, the lagged S&P500 dividend yield, the lagged slope of the term structure specified as the difference in yields of a 10-year Treasury bond and a 3-month Treasury bill, the lagged credit quality spread measured by the difference in yields of Moody's seasoned BAA-rated corporate bonds and AAA-rated corporate bonds, and an indicator variable for the month of January. We report results for a bootstrap simulation to determine the bias and efficiency of the CIOS, Ferson and Schadt (FS, 1996) and Christopherson, Ferson, and Glassman (CFG, 1998) conditional alpha estimates. We follow the bootstrapping approach described in Appendix 2, except we introduce abnormal performance into the regressand as follows, $\hat{R}_{2j,t} = abnorm_t + \hat{\phi}_{2j} R_{1,t} + \hat{\xi}_{2j,t}$

where the average abnormal return is generated as, $abnorm_t = \frac{0.3}{6} + \frac{0.3}{6\bar{Z}_1} Z_{1,t} + \frac{0.3}{6\bar{Z}_2} Z_{2,t} + ... + \frac{0.3}{6\bar{Z}_5} Z_{5,t}$.

We then calculate the *rmse* , *% rmse* , *bias* , and *% bias* for each estimation approach and each style portfolio for each of 5,000 bootstrapping simulations. The table reports the median value for the *rmse* , *% rmse* , *bias* , and *% bias* across the bootstrapping simulations for each estimation approach and style portfolio .

Portfolio and Alpha Source		*rmse*	*% rmse*	*bias*	*% bias*
Value	CIOS	0.0344	0.4585	0.0006	0.0002
	FS	0.0337	0.1509	0.0097	-0.1312
	CFG	0.0366	0.4690	0.0010	0.0039
		0.0176	0.2356	0.0002	0.0013
Blend	CIOS	0.0322	0.1290	0.0100	-0.1298
	FS	0.0184	0.2425	0.0010	0.0001
	CFG				
Growth	CIOS	0.1093	1.4546	0.0012	0.0093
	FS	0.0408	0.2616	0.0117	-0.1234
	CFG	0.1158	1.5156	0.0026	0.0112
Specialty	CIOS	0.0754	1.0121	0.0023	0.0051
	FS	0.0375	0.2104	0.0096	-0.1313
	CFG	0.0801	1.0383	0.0015	0.0012
Other	CIOS	0.0094	0.1277	-0.0000	0.0019
	FS	0.0314	0.1227	0.0082	-0.1369
	CFG	0.0101	0.1306	-0.0003	0.0015

Table 3
A simulation comparing the efficiency of various conditional alpha estimators

We generate data under the null hypothesis using the following model, $R_{2t} = a + bR_{1t} + cZ_t + \xi_{2t}$ with parameter values of $a = 0.15$, $b = 0.8$, and $c = 0.05$. The regressors R_{1t} and Z_t and the model disturbance, ξ_{2t}, are generated as $R_{1t} = 0.43 + 4.41rv_{1t}$, $Z_t = 1.72 + 1.13rv_{2t}$, and $\xi_{2t} = 0.5rv_{3t}$ where rv_{1t}, rv_{2t}, and rv_{3t} are random variables drawn from either a student t or normal distribution. We assume rv_{3t} is uncorrelated with rv_{1t} and rv_{2t}. Further, we consider correlations between R_{1t} and Z_t of 0, 0.5, and 0.9. Sample sizes of T=100, 200, and 1,000 are considered. The true conditional alpha is defined as $\alpha_{truet} = a + cZ_t$.

We then estimate equation (7), (8a), or (8b) for the CIOS, FS, or CFG models, respectively, and calculate the model specific conditional alpha, $\alpha_{est,t}$. The percentage root mean squared error ($\%\ rmse$) for any estimation approach is then defined as, $\%\ rmse = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\frac{\left(\alpha_{true,t} - \alpha_{est,t}\right)}{\alpha_{true,t}}}$. The above process is replicated for N=5,000 simulations. To compare the relative efficiency of the CIOS approach versus the FS approach, we calculate the percentage efficiency gain across replications as, $\frac{1}{N}\sum_{t=1}^{N}\frac{\left(FS_{\%rmse} - CIOS_{\%rmse}\right)}{CIOS_{\%rmse}}$. The related measure for the CFG estimator is constructed in an equivalent manner. The table reports the mean and median value for the percentage efficiency gain comparing the CIOS to either the FS or CFG approaches. A reported entry of 0.10 indicates that the CIOS estimator is ten percent more efficient than the alternative across all simulations. Means are reported first followed by medians in parentheses.

Panel A: Zero correlation results			
		Student t-distribution (df=4)	Normal distribution
		Means (medians)	Means (medians)
T=100	CIOS vs. FS	1.4480 (0.4077)	0.8298 (0.3207)
	CIOS vs. CFG	0.1756 (0.0088)	0.0571 (0.0026)
T=200	CIOS vs. FS	2.7512 (1.0050)	1.4938 (0.7267)
	CIOS vs. CFG	0.1672 (0.0059)	0.0288 (0.0006)
T=1000	CIOS vs. FS	10.1247 (3.9260)	4.4733 (2.7286)
	CIOS vs. CFG	0.1145 (0.0008)	0.0186 (0.0010)
Panel B: Large Sample (T=1000) – Non-zero correlations			
Correlation=0.5	CIOS vs. FS	8.1625 (3.3831)	3.9350 (2.3590)
	CIOS vs. CFG	0.1290 (0.0046)	0.0908 (0.0093)
Correlation=0.9	CIOS vs. FS	4.6119 (1.2986)	2.3008 (1.1197)
	CIOS vs. CFG	0.0860 (0.0021)	0.1057 (0.0111)