Systemic Risk as Renegotiation Breakdown: The Role of Structured Investment Products*

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August 2008

Abstract

We study the role of structured investment products (SIPs) in enabling banks to better hedge the risks in their asset streams but in generating greater systemic risk — the risk of financial distress spreading through the financial system — due to the linkages created by these contracts. By swapping out portions of their asset streams, banks lose the incentive to maintain the quality of their assets and compensate for lower quality with greater hedging. Banks attempt to renegotiate their SIP contracts ex post in the event of insolvencies at one or more banks to lower liquidation costs in the system. Renegotiations helps restore incentives but are unable to improve the equilibrium to the social optimum because they may break down and lead to inefficient liquidations. Breakdowns are endogenous in our model and occur in periods when several solvent banks are able to credibly threaten to 'run' from their obligations to weaker banks. The systemic transmission is greatest in periods of renegotiation breakdown. Pure interbank loans are ineffective in helping banks manage systemic risk, while asset swaps optimally chosen help increase social welfare and bank profits, lower credit risk, but increase the systemic risk of the system.

JEL Classification: G21, C1, C78, C81, E44

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Introduction

The explosion of the market for securitized assets in the form of Structured Investment Products (SIPs) that has facilitated the transfer of risk between financial institutions has been one of the more remarkable trends in finance. In contrast, the gross amounts of interbank loans, which can also be used for risk sharing between these institutions, has shown little growth during this period. Figure 1 shows the disparity in the growth of secured assets and interbank loans held at US banks. The recent increase in systemic concerns emanating from the defaults of subprime mortgage loans and resulting in perhaps the most severe banking crisis since the Great Depression. have left both academics and practitioners searching for rational economic explanations for both its severity and its rapid transmission. Many have explicitly put the blame on the huge mass of SIPs that has substantially changed the structure of financial markets as evidenced in the following comment:

> Wall Street got drunk. It got drunk and now it's got a hangover. The question is, how long will it sober up and not try to do all these fancy financial instruments? [President George W. Bush at a private political fundraiser in Houston, Texas, July 2008]

While it is a reasonable conjecture that the increase in financial linkages from the SIPs would increase the transmission channels by which shocks to one financial institution or sector can adversely affect other financial institutions thus generating systemic risk[1], taking stock of recent developments we highlight the elements of the current financial crisis that we build into an economic model to enhance an understanding of the role of SIPs in increasing the severity of the crises and its rapid transmission.

1. The SIPs have separated the origination and eventual ownership of the underlying financial assets and have reduced the incentives for originating institutions to maintain the quality of the assets originated.

2. Financial institutions attempt to renegotiate the payments on interbank SIP claims if one or more counterparties is insolvent to lower dead weight liquidation costs and increase recoveries. One means of insolvency management has been the explicit partial or complete mergers between banks that can be used to "net" out some of these claims.[2]

[1] While the exact definition of systemic risk remains up to debate [see Schwarcz (2007) for alternative definitions], one popular definition that has emerged is the risk that a default by one financial institution will have repercussions on other institutions due to the interlocking nature of financial markets. For example, a default by bank A on financial contracts on which it is due payments to bank B will affect affect the ability of B to come good on its obligations to bank C, and so on, in a cascading domino effect.

[2] The most prominent of the failures in the current crises was of Bear Sterns, which was eventually sold to J.P. Morgan in March 2008. Several other large institutions such as Citibank have been actively attempting to sell large parts of their assets to other institutions such as Sovereign Wealth Funds and better capitalized institutions in Asia.

3. Financial institutions in trouble often face liquidation even when they have a net positive value to the other banks in the system. This creates the need for a regulator to often coordinate the actions of the counterparties of the troubled bank.[3]

4. The rating agencies have been blamed for providing overly optimistic ratings for these SIPs, often ignoring the systemic component of risk over and above the inherent credit risks that are present in these contracts.

The goal of building a model with the above listed features is to ask several questions: Are SIPs effective vehicles for risk sharing, and if so, should they be more popular than interbank loans? In particular will they provide protection to distressed firms in their deepest crises? Will they lead to inefficient liquidation decisions of firms in distress? What sorts of SIPs lead to a larger chance of ineffective risk sharing and inefficient liquidation decisions? Does the asset transfer lower the quality of the underlying assets that are created by the financial institutions? Do profit maximizing banks transfer an efficient level of assets and does transfer lead to increased systemic risk? Is there a tradeoff between credit risk and systemic risk of financial institutions? Finally, does the bankruptcy regime in which the financial institutions operate in affect the credit and systemic risk created by SIPs?

In this paper we provide a theoretical model that provides a rationale for the popularity of SIPs and their advantage over interbanks loans for vehicles of sharing risks between financial institutions. The economic setting has N financial institutions (FIs), who could be traditional banks, investment banks, mortgage or finance companies, mutual funds, or hedge funds. We will simply refer to all such FIs as "banks". Each bank has ownership to a stream of assets, which in the credit risk literature is often referred to as the 'unlevered' asset value. The asset stream reflects the business of the bank in making loans and purchasing investments outside the banking system. It also has liabilities or deposits, which are senior to all other claims. To highlight the novelty of the economic mechanism introduced in this paper, we assume a two-period model, so that there are no short term liquidity problems that lead to bank runs as has been illustrated in a large literature starting with Diamond and Dybvig (1983) [Allen and Gale (2000) analyze systemic risk in such a setting]. Banks attempt to diversify the risk in their asset stream by engaging in interbank SIP transactions. SIPs are complex financial contracts that are made of simpler contracts such as pure loans, asset swaps, and possibly non-linear functions of their asset streams, such as tranches. These contracts help to smooth out the banks' profits. If at maturity of these contracts, all banks are solvent, then claims

[3] In the case of LTCM in 1998, the Federal Reserve Bank of New York stepped in and coordinated a bailout of the hedge fund by its counterparties.

are settled as contracted. However, if one or more banks are insolvent, we assume that these banks attempt to renegotiate the contractual terms of the contracts ex post to increase their recovery rate by avoiding liquidation costs. If renegotiations among the banks fail, then we assume that there is a well established bankruptcy code in the economy which determines how assets of the system of banks are divided among them when they have interbank SIP claims with each other. We describe this bankruptcy procedure next.

For standard debt claims, most papers assume that there is a bankruptcy code that maintains absolute priority and limited liability for the equity holders of the banks. Unlike the code for standard debt claims, the code for interbank claims must *simultaneously* solve for the a set of 'clearing' payments of each bank that satisfy a fixed point condition: receiving pro rata shares of these claims from all other banks, each bank is able to make the payment required of it. Note that imposing this procedure on the banks leads to systemic risk, as banks that are solvent but receive less than full payments on their interbank SIP claims from other banks are unable to make full payments on their commitments, and hence "pass on" their troubles to banks they must make payments to. The clearing system we use generalizes the work of Eisenberg and Noe (2001) by introducing liquidation costs. The framework has been used in Elsinger, Lehar, and Summer (2006) to determine the amount of systemic risk in a set of Austrian banks with interbank loans and a rich set of assets. Shin (2006) uses the framework to study the general equilibrium effects of increases and decreases in leverage.

The process of renegotiation is modeled as an N player noncooperative bargaining game that endogenously determines the recoveries on SIP contracts for all banks. Potential recovery on defaulting SIP contracts is higher and systemic risk is lower when banks that have positive equity but are unable to make all SIP payments, are *not liquidated* as the bankruptcy process defined above requires. However, if renegotiations among the banks fail, then the regulator steps in and imposes the bankruptcy code on the banks. Banks each attempt to maximize their personal recoveries by threatening to force other banks into the bankruptcy process. A significant contribution of our paper is to show that an efficient liquidation policy is not always possible as renegotiations are not always successful. The model reveals some interesting circumstances that lead to renegotiation breakdowns: In periods when an insolvent bank has SIP payments due to more than one solvent bank, each of the solvent banks is able to credibly threaten to 'run' with its due payment from the remaining banks because the latter find it in their interest to let the first mover run rather then to force the liquidation of the insolvent bank and obtain low recovery on their claims. There are cases when an insolvent firm can provide net equity to the collection of all solvent banks in the system, but no individual bank will bail it out since all the benefits of the bailout accrue to banks who do not join in the bailout. In this sense, the systemic runs from banks is a coordination failure among the solvent banks in the system, and is similar to the coordination failure among depositors in the bank model of Diamond and Dybvig (1983). However, our analysis is richer than the bank runs literature because we allow for possible mergers between the solvent banks to overcome the coordination failure. The lack of incentive compatibility of the efficient merging policy however leaves the coordination problem unsolved. [4] Our analysis shows that large systemic episodes are far more likely to happen due to renegotiation breakdowns than simply the periods in which banks all receive negative shocks.

The interaction in the model between hedging using SIPs and renegotiation is as follows: Banks must exert effort to increase the quality (mean of the asset distribution) of its asset streams. Swapping out a portion of the asset stream enables banks to share risks but leads to a reduction in the incentive of the bank to maintain an efficient level of effort. In a closed banking system with interdependent claims, the externalities of exerting effort are in part internalized as the credit quality of the counterparties of the banks, and hence the effectiveness of their SIP hedges improves with increased effort. However, this indirect benefit is realized with a probability (of the counterparty defaulting) that is smaller than one, so that in an equilibrium with SIP contracts, but no renegotiation, the effort level of all banks is far below its social optimum. Banks compensate for the lower quality of their assets by hedging more, but this reinforces the tendency to cut effort further. By modeling renegotiations, the benefit of each bank obtaining better recovery of its SIP claims increases, and their tendency to overhedge and underwork declines. However, renegotiations are not always successful, and limit the effectiveness of this channel, so that the level of effort with renegotiations are closer but yet short of the social optimum. Comparing equilibria in the economy with and without SIPs we find that banks profits and social welfare are higher in the presence of SIPs, the effect on credit risk among the banks is lower, and yet the systemic risk is invariably higher as financial distress spreads in states with renegotiation breakdown.

We next investigate the nature of SIP claims that can help optimize bank profits and the welfare of the system. Our first result is that pure interbank loans are relatively ineffective for hedging interbank risk relative to asset swaps, and the former would likely increase systemic concerns further. As we show, interbank loans do not improve the payoff of equity holders in periods when they are

[4] Major extensions of this bank run framework to study systemic risk due to liquidity shocks with general networks are in Allen and Gale (2000) (for a survey see Allen and Babus (2008)). As in our paper, Brusco and Castiglionesi (2007) introduce moral hazard issues to this framework. These papers also do not consider merging as a strategy for ex-post distress resolution as a means of solving the coordination problem.

solvent, and increase the risk of renegotiation breakdown situations, which occur when insolvent banks have large commitments to more than one counterparties. The inoptimality of interbank loans also helps distinguish the channels of systemic risk in our model from "liquidity shocks" at banks as modeled in Allen and Gale (2000). In their model, banks find it optimal to hold interbanks loans to better share the risks of such idiosyncratic liquidity shocks with other banks. In addition, the banks assets in their model are of long duration, and would not be useful in managing such risks. However, the relatively slow growth of such loans noted in Figure 1 suggest that the major risks at FIs in recent years are related to potential solvency problems as modeled in our paper, rather than liquidity shocks.

Specializing our analysis to a three bank system (the smallest number needed to generate renegotiation breakdowns) we find some interesting comparative statics. On average we find that banks optimally swap a small but positive proportion of their asset values. The internal solution provides a balance between the incentives for maintaining the quality of their assets and optimal interbank diversification. In economies in which asset values of banks are more highly correlated, the benefits of hedging are lower, and the adverse effect on incentives is lower. In such economies, effort and hedging levels are closer to the social optimum. In economies (or markets) with higher liquidation costs, we obtain the opposite effect, that is higher diversification needs and lower effort choices. Finally, in economies with weak bankruptcy regimes that imply a lower recovery in bankruptcy, all these above effects are adversely heightened if we do not model ex post renegotiations.[5] With ex post negotiations, surprisingly, the equilibria of the weak and strong bankruptcy regimes are very similar as successful renegotiations occur with about equal frequency. Ex post, banks stronger (weaker) banks are better off in the strong (weak) regime, but since banks are ex ante identical in our model, their overall welfare is about the same in the alternative regimes.

The paper also sheds light on the underestimation of credit risk by standard structural form models and the comovement of credit risk across firms. Huang and Huang (2003) show that once standard credit risk models are calibrated to match historical leverage ratios, volatilities, and default probabilities of alternative rating categories, they predict credit spreads much smaller than those observed historically. On a similar note Collin-Dufresne, Goldstein, and Martin (2001) show that credit spreads of different firms, both financial and nonfinancial, seem to have excess comovement after accounting for the leverage ratios of firms, and their asset volatilities (together comprising the distance-to-default) of firms. Therefore, measures of credit risk seem to be more correlated

[5] Weak and strong bankruptcy regimes are identified as alternative fixed points of the clearing vector described above that have the minimum and maximum payments by all banks.

than the inputs of the structural form model of credit risk. Our model provides an explanation of both findings for financial institutions. Calibrating the models to a firm's capital structure without accounting for the systemic risk from off balance sheet SIP linkages will underestimate the credit risk of the bank and its correlation with troubles at other banks. Default probabilities and correlation of bank liquidations in our model is very close to the correlation of their asset streams if there is no interbank hedging but are significantly higher once we model systemic risk with renegotiation breakdowns. The large systemic concern that arises from credit risk transfer in our model also sheds light on the underestimation of credit risk of structured investment products by the rating agencies.

Related Literatures

While market participants and policy makers have been concerned about systemic risk, the limited empirical work on the topic has found it to be a very low probability risk. Earlier simulation studies analyzing interbank exposures such as Humphrey (1986), Angelini, Maresca, and Russo (1996), Sheldon and Maurer (1998), Furfine (2003), Degryse and Nguyen (2004), Wells (2002), and Upper and Worms (2004) investigate contagious defaults that result from the hypothetical failure of a single institution. These papers take a given set of interbank exposures, assume some financial institutions to default and then mechanically clear the interbank market and record which banks are dragged into insolvency. Such analyses are able to capture the effect of idiosyncratic bank failures (e.g., due to fraud). This approach can be seen as isolating one source of systemic risk, namely, interbank linkages and ignoring the other: correlation in the banks' exposures. Elsinger, Lehar, and Summer (2006) study the credit risk in the Austrian Banking system. They model macroeconomic shocks that hit all banks loan and trading portfolios simultaneously. When defaults occur they analyze how they propagate through the network of interbank exposures and find that the correlation of bank exposures more relevant to generating multiple defaults, than contagion.

Our paper also contributes to the literature on the renegotiation of debt contracts. In most papers a solution to the bargaining game at the time of renegotiation always exists due to the special assumptions made in these papers. Several papers assume that players are able to make "take-it-or-leave-it offers" with exogenous bargaining strengths [see, e.g. Hart and Moore (1998), Garleanu and Zwiebel (2006), and Hackbarth, Hennessy, and Leland (2007)]. Paper such as Bolton and Scharfstein (1996), Rajan and Zingales (1998) and David (2001) endogenize bargaining power using the Shapley value of the game as the solution concept. However, these papers make enough assumptions so that all papers agree to renegotiate with each other and the 'grand coalition' forms. Our work is motivated on recent work on bargaining with externalities by Maskin (2003) who argues that in many real world situations the grand coalition fails to form but a subset of the players agree to divide resources.[6] As in Maskin (2003), we assume the sequential random arrival of banks to a bargaining site where not only the division of the pie but the decisions by banks on who to bargain with is endogenously determined. The random arrival order takes away any first mover advantage to any given bank.

The empirical investigation of renegotiations is still at an early stage but recent work suggests that it is critical to include renegotiations among counterparties to evaluate the effects of financial distress. Roberts and Sufi (2007) provide an empirical analysis of the renegotiation of private credit agreements between US public firms and financial institutions. They report that over 90 percent of long-term debt contracts are renegotiated prior to maturity. However, their method of data collection does not pick up failed renegotiations.

Finally, systemic risk, the risk of insolvencies spreading through the financial system due to interlocking financial contracts, is similar to a related literature on contagion, that also potentially explains why there is a high correlation of financial distress across markets and firms. Pritsker and Kodres (2002) find learning and hedging effects across markets can cause contagion (comovement) in financial markets. Acharya and Yorulmazer (2006) use such an information contagion mechanism to study correlation in bank failure. Collin-Dufresne, Goldstein, and Helwege (2003) use a learning mechanism to explain why credit spreads across all firms increased sharply after the revelations of financial trouble at Enron in 2001. We note, that the channel outlined in this paper to generate correlation in financial distress relies on valuation effects of interbank contracts and holds in a setting with complete information.

The remainder of the paper is structured as follows: In section 1, we provide the structure of the model and the bankruptcy procedure that settles claims in an interbank system. In section 2, we provide a game theoretic analysis of renegotiations among bank, and in section 3 we study optimal SIP contracts. Section 4 concludes. An appendix provides the technical details of an algorithm that solves for values of all banks in a renegotiation for the general case of N banks.

[6] There are several prominent examples of incomplete participation agreements such as the European Union and the Kyoto Protocol.

1 The Model

We consider a simple two period model of a banking system. All contracts are written at date 0 and are settled at date 1.

Assumption 1: There are N identical risk neutral banks, each of which has an 'outside' random asset value, $\tilde{A}_i$. For simplicity we assume that the N asset distributions are identically distributed

$$A_i \sim \text{LN}(\mu_0 + \mu_1 \, h_i - 0.5 \, \sigma^2, \sigma),$$

where the assumption of log-normality has no special purpose expect to ensure that the assets always have positive value. Each asset value has a correlation of ρ with each other asset payoff. The term h_i represents the level of effort that each bank can exert to increase the mean of the asset value. We assume that the effort has a cost to each bank of $\gamma \, h_i^2$. The effort is financed by the equity holders and the cost is incurred at date 0. At date 1, this is sunk cost, and hence does not affect settlements.

Assumption 2: Each bank has a senior deposit liability payment due at maturity of L_i. The equity of each bank is $\tilde{e}_i = \tilde{A}_i - L_i$ We assume that all depositors are risk-neutral and have zero time discount. This deposits are senior to all other claims made by the banks. Each bank purchases fairly priced deposit insurance for its deposits. The deposit insurance premium is determined by

$$\omega_i^D = E\left[\mathbf{1}_{\{D_i > 0\}} \, \max[L_i - (1 - \Phi)\tilde{A}_i - r(\{i\}), 0]\right], \forall i \in \mathcal{N} \tag{1}$$

where $\mathbf{1}_{\{D_i>0\}}$ is a liquidation indicator for bank i, which takes the value of 1 whenever the assets of the bank are liquidated, a fraction Φ of the assets are lost upon liquidation because of bankruptcy costs, and $r(\{i\})$ is the payment received by bank i on its interbank claims. The deposit insurance premium is also financed by the equity holders and the cost is incurred at date 0. As the effort costs above, at date 1, this cost is sunk cost as well, and hence does not affect settlements.

Assumption 3: Each bank enters into interbank risk sharing agreements with each other bank, each promising a state contingent payoff of

$$\tilde{l}_{ij} = a + b \max[\tilde{A}_i - k, 0], \quad \forall i, j \in \mathcal{N}.$$

The interbank claims are junior to the deposits. Note that the first part, a represents a pure interbank loan, since it offers a fixed payment. The loan is risky since the bank may not be able to repay it in full. The second part, is an asset swap that each bank i offers to swap out to each bank j an amount $b \max[\tilde{A}_i - k, 0]$ of its asset flow, in return for the same amount of $\max[\tilde{A}_j - k, 0]$. Notice that the *quid pro quo* exchange arises from the assumption that the banks are ex-ante identical so that the flows have equal discounted values. Also note that since the ex-ante values of the total payments are identical, entering into such agreements has no impact on the leverage ratios of the banks. When $k = 0$, the second part represents a plain vanilla swap, while for $k > 0$, it represent the upper tranche of an asset backed security. We refer to the contracts $\tilde{l}_{ij}$ as structured investment products (SIPs).

The structure of this network of banks for the case where $N = 3$ is displayed in Figure 2. We will study the optimal ex-post settlement policy of the banks of their deposits as well as their interbank claims.

Assumption 4: The decisions of the banks are made by their equity holders. At the time of settlement of these claims the banks may consider 'mergers' that essentially net out their outside investments and interbank claims Therefore a coalition of banks S has outside assets of $\tilde{A}(S) = \sum_{i \in S} \tilde{A}_i$, liabilities of $L(S) = \sum_{i \in S} L_i$, and interbank claims of $\tilde{l}_{Sj} = \sum_{i \in S} \tilde{l}_{ij}$, and $\tilde{l}_{jS} = \sum_{i \in S} \tilde{l}_{ji}$ for any $j \notin S$.

Assumption 5: The bank faces liquidation costs that are a fraction Φ of the ex-post value of its assets if the equity holders of the bank decide to liquidate the assets.

Assumption 6: At date 1 the N banks attempt to settle all claims. If all banks are solvent ex-post, then all claims are settled in full. Otherwise, the N banks attempt to renegotiate these claims and decide on which banks should be optimally liquidated. If renegotiations break down then we assume that a regulator imposes the bankruptcy code of the economy on these banks, which determines how claims are settled. For the banking system with interbank claims, the division of assets of each bank poses a simultaneous system of conditions, since the amount each bank can pay the other banks depends on how much it receives from these other banks. We call such a system a *clearing* vector, which we describe in detail in section 1.1 below.

1.1 Determination of Clearing Vectors

If at date 1, the banks are unable to settle all claims, then the regulator of the economy steps in and determines a clearing vector of payments that each bank in the system makes in lieu of its promised payments. We generalize the seminal analysis of clearing vectors in Eisenberg and Noe (2001) to include liquidation costs. In addition, we analyze the clearing vectors when banks consider mergers as in Assumption 5 to resolve all financial claims before proceeding to the regulator for a resolution of claims. We denote the complete set of banks with the set $\mathcal{N} = \{1, ..., N\}$. If the banks consider mergers then the resulting set is $\mathcal{F} = \{1, ., f, .., F\}$, where $f \in \mathcal{F}$ consists of one or more merged banks in $\mathcal{N}$, and is $\mathcal{F}$ is thus a "partition" of the original set $\mathcal{N}$. When modeling renegotiations among different banks in the following sections, we will analyze the strategy of various merged banks. Here we will characterize the clearing vector for the general partition $\mathcal{F}$, which for the special case that all banks approach the regulator without merging leads to the clearing vector for the original set $\mathcal{N}$. We therefore generalize the notation of Eisenberg and Noe (2001) to include the superscript $\mathcal{F}$ to denote that the clearing vector is conditional on the banks' merging strategies.

Let $\tilde{d}^{\mathcal{F}}(\{i\}) = \sum_{j=1}^{F} \tilde{l}_{ij}^{\mathcal{F}}$, be the total obligations of the merged bank i in the partition $\mathcal{F}$. We define the relative liabilities matrix of the partition $\mathcal{F}$ as $\tilde{\Pi}^{\mathcal{F}}$ with elements

$$\begin{aligned}\tilde{\Pi}_{ij}^{\mathcal{F}} &= \frac{\tilde{l}_{ij}^{\mathcal{F}}}{\tilde{d}^{\mathcal{F}}(\{i\})} \quad \text{if } \tilde{d}^{\mathcal{F}}(\{i\}) > 0 \\ &= 0 \qquad\qquad \text{otherwise.}\end{aligned}$$

Let $\tilde{p}^{\mathcal{F}}$ be the F vector of payments that each bank makes. Then, the vector of clearing payments received by the banks are given by the vector $\tilde{r}^{\mathcal{F}} = (\tilde{\Pi}^{\mathcal{F}})' \cdot p^{\mathcal{F}}$. Then the clearing vector $p^{\mathcal{F}}$ for this banking system must satisfy

$$\tilde{p}^{\mathcal{F}}(\{i\}) = \min\left[d^{\mathcal{F}}(\{i\}), \max\left(\tilde{A}_i^{\mathcal{F}} - \Phi \tilde{A}_i^{\mathcal{F}} \mathbf{1}_{\tilde{p}^{\mathcal{F}}(\{i\})<d^{\mathcal{F}}(\{i\})} + \tilde{r}^{\mathcal{F}}(\{i\}) - L_i^{\mathcal{F}}, 0\right)\right], \forall i \in \mathcal{F}. \tag{2}$$

The definition states that either bank i makes its full interbank payment of $d^{\mathcal{F}}(\{i\})$, or the regulator will liquidate its assets with a proportional liquidation cost of Φ and these proceeds are used along with the payments that i receives from the other banks to first pay off the deposit holders, and the remaining amount is paid to the other banks in settlement of its interbank claims. This can be written more compactly as

$$\tilde{p}^{\mathcal{F}} = \min\left[\tilde{d}^{\mathcal{F}}, \max\left[\tilde{A}^{\mathcal{F}} - \Phi\, \tilde{A}^{\mathcal{F}} \mathbf{1}_{\tilde{p}^{\mathcal{F}}<d^{\mathcal{F}}} + (\tilde{\Pi}^{\mathcal{F}})' \cdot \tilde{p}^{\mathcal{F}} - L^{\mathcal{F}}, 0\right)\right], \tag{3}$$

where $\max$, $\min$, and $\mathbf{1}$ denote the component wise maximum,minimum, and indicator functions respectively. The right hand side of this equation can be written as a vector valued mapping $\Psi(\tilde{p})$. Stated alternatively, the clearing vector is the fixed point of this mapping. It is straightforward to show by Kakutani's fixed point theorem that there is at least one fixed point of this mapping. As in

Eisenberg and Noe (2001), we will find it by the method of successive approximation, which these authors call the ‘fictitious default’ algorithm.

Besides establishing existence, Eisenberg and Noe (2001) also provided conditions under which the fixed point of the mapping is unique for the case of zero liquidation costs. We instead find a robust set of examples with positive liquidation costs in which there are two at least two fixed points. We first provide an example and then an interpretation of the se two fixed points as alternative bankruptcy regimes.

Example 1 (Non Uniqueness of Clearing Vectors)

Consider the case of three banks that have ex-post asset values, $\tilde{A}_i$ of $1.011, 0.972$, and 1.048, for i = 1,2, and 3, respectively. Each bank has deposits of 1. Bank 1 owes banks 2 and 3 0.403 each, bank 2 owes 0.391 each, and bank 3 owes 0.414 each. Proportional liquidation costs $\Phi = 0.1$. Then there are two clearing vectors. In the first, the payments made by the banks to the other banks are $\{0.806, 0.782, 0.828\}$, that is, each bank makes a full payment. It is easily verified that with these payments, each bank is able to make its full payments to all depositors and banks, and is not liquidated. For example, bank 2 receives $0.403 + 0.414 = 0.817$, so that its total resources available for distribution are $0.972 + 0.817 = 1.789$. Its total commitments are 1.782, so it does not face liquidation.

The other clearing payment vector is $\{0, 0, 0\}$, that is no bank makes any payments to other banks or receives anything. Now all three banks are insolvent, and their assets are liquidated. For example, consider the first bank. Since it receives nothing, it has total assets of 1.011 and commitments of 1.806, hence it is liquidated. After liquidation, it has $0.9 \cdot 1.011$ available for distribution, which equals 0.9099. This is clearly smaller than the 1 it owes its depositors. So, its pays 0 for all its interbank commitments. The same happens to the other banks. Notice the “systemic” risk in this clearing payment vectors. Each bank defaults on its commitments only because it receives nothing on commitments owed to it. We will make this definition more precise below.

The role of a non-zero Φ is important. If $\Phi = 0$ then as in Eisenberg and Noe (2001), we would have a single clearing vector, the first one. For any $\Phi > 0.01$ though, the second clearing vector is also valid.

Finally, its worth pointing out that the example is a little extreme because with the second clearing vector all payment vectors are zero. We can construct similar examples where only one or two banks have zero clearing payments.

Motivated by the example and following Elsinger, Lehar, and Summer (2006) we make a distinction between 'fundamental" defaults, and "contagious" defaults. The default of bank i is called fundamental if bank i is not able to honor its promises under the assumptions that all other banks honor their promises,

$$\sum_{j=1}^{F} \tilde{\Pi}_{ji}^{\mathcal{F}} d^{\mathcal{F}}(\{j\}) + \tilde{e}_i^{\mathcal{F}} - d^{\mathcal{F}}(\{i\}) < 0.$$

A contagious default occurs, when bank i defaults only because other banks are not able to keep their promises, i.e.,

$$\sum_{j=1}^{F} \tilde{\Pi}_{ji}^{\mathcal{F}} d^{\mathcal{F}}(\{j\}) + \tilde{e}_i^{\mathcal{F}} - d^{\mathcal{F}}(\{i\}) \quad \geq \quad 0 \tag{4}$$

$$\text{but} \tag{5}$$

$$\sum_{j=1}^{F} \tilde{\Pi}_{ji}^{\mathcal{F}} p^{\mathcal{F}}(\{j\}) + \tilde{e}_i^{\mathcal{F}} - d^{\mathcal{F}}(\{i\}) \quad < \quad 0. \tag{6}$$

Using these definitions, the defaults in the second clearing vector are all contagious.

We interpret the two different clearing vectors as two distinct bankruptcy regimes. The first we call the "strong" regime, since it implies that all banks pay larger amounts for their interbank commitments, and in turn receive more from other banks. The second is the "weak" regime, in which banks pay out less and receive less on their commitments. Both clearing vectors are 'fair' in the sense that limited liability of all equity holders and absolute priority of all claims is maintained in both. The choice of the regime is determined by the enforcement power of the regulator, and its determination is outside the scope of this model. However, we note that unlike the analysis in Eisenberg and Noe (2001) and Elsinger, Lehar, and Summer (2006), we do not assume that banks actual payments for their interbank claims are determined completely by the clearing payment vectors. The clearing vector is the value that each bank will pay out if the set of banks jointly fail to renegotiate all claims among themselves, and approach the regulator. In the next section we model these renegotiations and then study the implications for recovery rates on the interbank claims in the two bankruptcy regimes.

2 Renegotiation of Interbank SIP Payments

In this section we provide an analysis of the bargaining game that takes place at date 1 between the N banks if some or all of them fail to make full payments on the interbank SIP commitments. The banks consider mergers with each other for the resolution of their claims.

2.1 Externalities and Games in Partition Form

In considering the strategies of banks if they decide to merge, we must consider the value that can be realized by a coalition of banks. Games that start with the specification of values of coalitions are called characteristic function games. In such games a coalition S of the set of players can obtain the payoff $v(S)$ irrespective of the actions of other players.[7] In contrast, in this paper the value that a coalition S can obtain depends on the actions and the merging strategies of other banks. Let the merging strategies of the full set of players lead to a partition $\mathcal{F}$ of the set of all players. Then, we will write the value of a coalition of banks S as $v^{\mathcal{F}}(S)$ to denote the value that this set of banks can attain when playing in the partition $\mathcal{F}$. In general the value will be different when different partitions are formed. In particular we are interested in what happens to the value of S when two banks in the partition $\mathcal{F}$ merge. To be formal, let F_1 and F_2 denote two coalitions in the partition $\mathcal{F}$. Let $\mathcal{F}_{12}$ denote the partition that is formed when the two banks F_1 and F_2 in $\mathcal{F}$ merge, and all other banks remain the same. Following Maskin (2003) we say that the externality from the merger is positive if for some coalition of banks $F \notin (F_1 \bigcup F_2)$ in $\mathcal{F}$,

$$v^{\mathcal{F}}(F) < v^{\mathcal{F}_{12}}(F). \tag{7}$$

Therefore, the value of the coalition F increases when coalitions F_1 and F_2 merge.

Such positive externalities naturally arise in financial applications. Think of an insolvent borrower with two well capitalized creditors. By merging with the insolvent borrower one creditor can bail it out and thus reduce potential liquidation costs. The merger however creates a positive

[7]Probably the most famous solution of characteristic function games is the Shapley value, which assigns the average marginal product to each player and satisfies very reasonable axioms of a bargaining process. For games with a well defined characteristic function, Hart and Mas-Colell (1996) provides an explicit non-cooperative alternating offers game in which the N players agree to assign each player her Shapley value. It will be evident from our analysis that the solution of our game will be the Shapley value when the endogenously chosen partition by the banks is the grand coalition but will differ otherwise.

externality for the other creditor, who can collect its full promised payment. The value that this second lender can achieve clearly depends on the strategy of the borrower and the first lender. [8] The existence of positive externalities implies that that an efficient outcome will not always be achieved. In the example above, one bank has an incentive to free ride and hope that the other bank will bail out the failed institution. We will show more detailed examples of an inefficient outcome later after we have defined the rules of the bargaining game.

2.2 The Bargaining Protocol

The game starts with nature choosing an order of the banks at which they will arrive at the bargaining site. The order is maintained in two important stages of the game: (i) A bank higher in the order gets to bid for the claims of all banks lower in the order, and (ii) If two banks remain independent, and they both bid for a third bank, then a bank higher in the order places the earlier bid. Conditional on the randomly picked order, banks make take-it-or-leave-it offers, that is the bank receiving the order can simply reject the bid or fail to reject it and compare the bid with competing bids from other banks.[9]

The game proceeds as follows: The first player forms a singleton coalition. Each later player n that arrives, faces a partition, i.e. a collection of coalitions $1, \cdots, F$. Coalition 1 makes a bid for bank n, which is an offer of a cash payment that n gets in exchange for signing up with the coalition and surrendering all its claims. Bank n either rejects the offer (in which case we assume that coalition 1 and bank n can never be in the same coalition again), or it fails to reject the bid, and then it can entertain bids from coalitions $2 \cdots F$, and pick the highest bid. As we will see, the optimal bidding strategy of the bank with the highest valuation is to bid the maximum of bids of the other banks' valuations. Joining a coalition is a binding merger agreement, i.e., player cannot leave a coalition at a later stage of the game. If bank n rejects all bids, it will remain independent, i.e. it will be in a singleton coalition. Once all players are assigned to coalitions, the final partition is determined and payoffs for coalitions are realized. The players who joined a coalition receive the payoff they were promised upon signing up, and the payers that started a coalition keep the payoff of the coalition minus the payments that they promised to the other coalition members. Since the set of bank mergers and hence the resulting eventual partition of players is endogenous, we will keep

[8] Externalities are present in a lot economic problems where coalitions are formed. Consider, for example, cartels, where the price increase due to output reduction of the cartel members benefits firms that choose not to join the cartel. For an excellent summary of the recent literature see Ray (2007).

[9] Our choice of these sequential rules of bargaining is motivated by the work of Maskin (2003) who shows that with such rules there is a larger set of circumstances in which players will find a bargaining solution relative to a game where players can make simultaneous offers. See in particular Example 1 of his paper.

a track of both players' payoffs (by the function ϕ) and the partition (by the function ψ) at each node of the tree. Since there are potential gains from all banks cooperating and hence minimizing liquidation costs we will say that *renegotiations break down* whenever the grand coalitions of all banks do not form.

The clearing vector of the economy determines the reservation values for banks as they evaluate bids to be taken over. If there are no merger agreements each bank can obtain a minimum payoff of $v^{\mathcal{N}}(\{i\})$ that satisfies

$$v^{\mathcal{N}}(\{i\}) = \max[\tilde{e}_i + r^{\mathcal{N}}(\{i\}) - \tilde{d}^{\mathcal{N}}(\{i\}), 0]. \tag{8}$$

Note that $v^{\mathcal{N}}\{i\})$ is completely determined by the contractual ex-post payment the bank is obliged to make and the clearing vector that is enforced by the bankruptcy regime. Similarly, we can define the minimal threat points of various subsets of banks, when they are in a game with a partition $\mathcal{F}$. Then the payoff that the subset $S \in \mathcal{F}$ obtains without renegotiation is

$$v^{\mathcal{F}}(S) = \max[\sum_{i \in S} \tilde{e}_i + r^{\mathcal{F}}(S) - \tilde{d}^{\mathcal{F}}(S), 0]. \tag{9}$$

2.3 Solving for Equilibrium of the Two Bank Case

We first provide an analysis for the two bank case where as we see the equilibrium of the bargaining game always leads to an efficient liquidation policy. The analysis could be for an economy with only two banks, or for a subgame with two merged banks from a larger banking system. We simply write all payoffs conditional on the current filtration being $\mathcal{F} = \{1, 2\}$, which could arise from either of both these situations. For the two bank case, the clearing payment vector in (2) can be written more simply as

$$p^{\mathcal{F}}(\{i\}) = \min\left[\tilde{l}_{ij}^{\mathcal{F}}, \max\left(A_i^{\mathcal{F}} - \Phi A_i^{\mathcal{F}} \mathbf{1}_{p^{\mathcal{F}}(\{i\})<d_i^{\mathcal{F}}} - L_i^{\mathcal{F}} + p^{\mathcal{F}}(\{j\}), 0\right)\right], \tag{10}$$

for i = 1,2, and $j \neq i$. and the clearing receiving vector is $\{r^{\mathcal{F}}(\{1\}), r^{\mathcal{F}}(\{2\})\} = \{p^{\mathcal{F}}(\{2\}), p^{\mathcal{F}}(\{1\})\}$, since all the payments made by bank i are received by bank j. We now provide simple conditions under which it is optimal (efficient) to liquidate one or more banks.

Result 1 *In the two person game the necessary and sufficient efficient conditions under which at least one bank is liquidated are as follows: Either* $\tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}} < 0$ *or* $\tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}} > 0$ *and for* $i \neq j$

$$\tilde{e}_i^{\mathcal{F}} + d^{\mathcal{F}}(\{j\}) \quad < \quad 0 \quad \textit{and} \tag{11}$$

$$\tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) \quad > \quad 0, \tag{12}$$

in which case bank j *will force a liquidation of the assets of bank* i*. Apart from these two cases,* $v^{\mathcal{F}}(\{1,2\}) = \tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}}$*.*

The optimal liquidation policy ensures that the value of combining the claims of two banks cannot be smaller than the value of each bank alone. If one of the banks has equity so far negative that even after receiving its interbank contractual clearing payment, its value is still negative, then the two bank coalition will optimally liquidate this bank, and share the resources of the other bank.

Result 2 *The two bank bargaining game always leads to efficient liquidations as in Lemma 1. In addition, the order of proposers is irrelevant.*

The intuition as for the bargaining equilibrium leading to an efficient liquidation policy is that if the solvent bank decides to not liquidate the bank in trouble, it can fully appropriate the preempted liquidation costs. We shall see in the following subsections when there are three banks this result will no longer hold. It is also useful to note that for the case that both banks remain solvent, there is no benefit to the banks from merging since the reservation values of the banks equal their values in solvency. Thus our model has no implications for bank mergers over and above their role in resolving financial distress.

We now illustrate with a simple example why replacing the condition (12) by the weaker one: $\tilde{e}_j^{\mathcal{F}} - p^{\mathcal{F}}(\{j\}) > 0$ will not provide a sufficient condition for liquidating the insolvent bank.

Example 2:

Let $A_1 = 2$, $A_2 = -0.7$, and $L_i = 1$, for $i = 1, 2$. Let $\Phi = 0.4$. Then bank 1 makes a payment of $p_1 = 0.6\,2 - 1 = 0.2$, while bank 2 pays 0 on its SIP commitments. Then $e_1 - p_1 = 0.8 > 0$, but if bank 1 forces a liquidation of bank 2, its equity holders keeps a profit of 0 for themselves, since all its assets are exhausted meeting the SIP commitments. Merging the banks however gives the combined shareholders positive equity of 0.7, which by Result 2 can all be appropriated by the equityholders of bank 1.

2.4 Solving for Equilibrium of the Three Bank Case

In the three player case we can analyze the game in greater detail. Figure 3 shows the extensive form of the three player game. Expected payoffs are obtained by averaging over the orders. Denote by b_i^X the bid that bank i makes at node X and let $\bar{b}_i^X$ be the maximum that bank i is willing to bid at node X.

Result 3 *The solution of the three player bargaining game for the natural arrival order, i.e.,* $1, 2, 3$ *given in Figure 3 is as follows:*

(i) Conditional on the game reaching node E, bank 2's maximum bid $\bar{b}_2^E$ *is*

$$\bar{b}_2^E = v^{\{1\},\{2,3\}}(\{2,3\}) - v^{\mathcal{N}}(\{2\}). \tag{13}$$

(a) If $b_2^E = \bar{b}_2^E$*, then* $\phi^E = \left(v^{\mathcal{N}}(\{1\}), v^{\mathcal{N}}(\{2\}), v^{\mathcal{N}}(\{3\})\right)$ *and the realized partition is* $\psi^E = \mathcal{N} = \{\{1\},\{2\},\{3\}\}$.

(b) Otherwise $\phi^E = \left(v^{\{\{1\},\{2,3\}\}}(\{1\}), v^{\{\{1\},\{2,3\}\}}(\{2\}) - v^{\mathcal{N}}(\{3\}), v^{\mathcal{N}}(\{3\})\right)$ *and the realized partition is* $\psi^E = \{\{1\},\{2,3\}\}$.

(ii) Conditional on the game reaching node C, bank 1's maximum bid for bank 3 is

$$\bar{b}_1^C = v^{\{1,3\},\{2\}}(\{1,3\}) - \phi_1^E, \tag{14}$$

and if bank 1's bid is rejected, the game moves to node F and bank 2's maximum bid is

$$\bar{b}_2^F = v^{\{1\},\{2,3\}}(\{2,3\}) - v^{\{\{1,3\},\{2\}\}}(\{2\}). \tag{15}$$

At node E, the payoffs and realized partitions are

(a) If $\bar{b}_1^C < v^{\mathcal{N}}(\{3\})$, $\phi^C = \phi^E$*, and the realized partition is* $\psi^C = \psi^E$ *as described in (i),*

(b) If $v^{\mathcal{N}}(\{3\}) < \bar{b}_1^C$ *and* $\bar{b}_2^F < \bar{b}_1^C$*,*

$$\psi^C = \left(v^{\{1,3\},\{2\}}(\{1,3\}) - \max(v^{\mathcal{N}}(\{3\}), \bar{b}_2^F), v^{\{1,3\},\{2\}}(\{2\}), \max(v^{\mathcal{N}}(\{3\}), \bar{b}_2^F)\right),$$

and the realized partition is $\psi^C = \{\{1,3\},\{2\}\}$.

(c) If $v^{\mathcal{N}}(\{3\}) < \bar{b}_1^C < \bar{b}_2^F$ $\phi^C = \left(v^{\{1\},\{2,3\}}(\{1\}), v^{\{1\},\{2,3\}}(\{2,3\}) - \bar{b}_1^C, \bar{b}_1^C\right)$*, and the realized partition is* $\psi^C = \{\{1\},\{2,3\}\}$.

(iii) Conditional on the game reaching node D, bank 1's maximum bid for bank 3 satisfies:

$$\bar{b}_1^D = v^{\{1,2,3\}}(\{1,2,3\}) - v^{\{1,2\},\{3\}}(\{1,2\}), \tag{16}$$

and the realized payoffs and partitions are

(a) If $b_1^D = \bar{b}_1^D$, $\phi^D = \left(v^{\{1,2\},\{3\}}(\{1,2\}) - b_1^B, b_1^B, v^{\{1,2\},\{3\}}(\{3\})\right)$ *, and the realized partition is* $\psi^D = \{\{1,2\},\{3\}\}$*,*

(b) Otherwise $\phi^D = \left(v^{\{1,2,3\}}(\{1,2,3\}) - b_1^B - v^{\{1,2\},\{3\}}(\{3\}), b_1^B, v^{\{1,2\},\{3\}}(\{3\})\right)$ *, and the grand coalition is realized* $\psi^D = \{1,2,3\}$*.*

(iv) As node B, the node where bank 1 first makes a decision:

(a) If $b_1^B = \phi_2^C$ $\phi^B = \phi^D$*, and the partition* $\psi^B = \psi^D$ *as defined in (iii) will be realized,*

(b) Otherwise $\phi^B = \phi^C$*, and the partition* $\psi^B = \psi^C$ *as defined in (ii) will be realized.*

The ex ante payoffs for the players ϕ *are determined by averaging* ϕ^B *over all possible arrival orders.*

2.4.1 Example: Renegotiation Inefficiency

To see how externalities can cause inefficient liquidations, consider the example illustrated in Figure 4.The outside liabilities L for each bank are assumed to be 1 and proportional liquidation costs are $\Phi = 0.3$. Bank 3 is in fundamental default. Even when it can collect all the promised payments from the other banks of 0.2, it cannot meet its interbank obligations and has net value of $A_1 - L + l_{13} + l_{23} - l_{31} - l_{32} = 0.895 - 1 + 0.1 + 0.1 - 0.1 - 0.1 = -0.105$. Thus, if no renegotiations occur, bank 3 is liquidated, reducing its asset value to $A_3(1 - \Phi) = 0.895(1 - 0.3) = 0.6265$. Even when bank 3 collects all interbank claims, it has $0.6265 + 0.2 = 0.8265$ which is less that its outside liabilities L and bank 3 shareholders as well as the other banks receive zero. Banks 1 and 2 are well enough capitalized to survive.

For each possible partition we can then compute the clearing vector according to Equation (3) and the value of the equity holders' claim without renegotiations using Equation (9).

Coalition structure	Payoff equityholders	Comments
{1,2,3}	$v^{\{1,2,3\}}(\{1,2,3\}) =$ 0.115	$A_1 + A_2 + A_3 - 3L$
{1,2},{3}	$v^{\{1,2\},\{3\}}(\{1,2\}) =$ 0.02	
	$v^{\{1,2\},\{3\}}(\{3\}) =$ 0	Bank 3 fails
{1,3},{2}	$v^{\{1,3\},\{2\}}(\{1,3\}) =$ 0.005	
	$v^{\{1,3\},\{2\}}(\{2\}) =$ 0.11	
{1},{2,3}	$v^{\{1\},\{2,3\}}(\{1\}) =$ 0.11	
	$v^{\{1\},\{2,3\}}(\{2,3\}) =$ 0.005	
$\mathcal{N}$={1},{2},{3}	$v^{\mathcal{N}}(\{1\}) =$ 0.01	
	$v^{\mathcal{N}}(\{2\}) =$ 0.01	
	$v^{\mathcal{N}}(\{3\}) =$ 0	Bank 3 fails

There are three possible partitions that maximize welfare, i.e. the sum of the players' payoffs: the grand coalition $\{1,2,3\}$ and the two partitions in which one of the financially sound banks bails out the troubled bank $\{1,3\}$, $\{2\}$ and $\{1\}$,{2,3}. As we will see below, the welfare maximizing outcome cannot be realized in all cases, because an individual bank is better off liquidating a troubled bank than bailing it out. If Bank 1 bails out bank 3, it can at most get $v^{\{1,3\},\{2\}}(\{1,3\}) = 0.005$, whereas it can get $v^{\mathcal{N}}(\{1\}) = 0.01$ if bank 1 is liquidated.

In the discussion we follow the extensive form and refer to nodes as labeled in Figure 3. Which partition will be realized depends on the order of arrival. Consider tow representative cases of arrival orders

Arrival order 1 2 3: Consider the subgame in node E first: Banks 1 and 2 are separate and Bank 3 has rejected Bank 1's bid. Bank 2 has to decide how much to bid for Bank 3. By staying independent Bank 2 will get $v^{\mathcal{N}}(2) = 0.01$ whereas it could get at most $v^{\{1\},\{2,3\}}(\{2,3\}) = 0.005$ from signing up 3. Bank 2 will therefore make an infeasible bid for Bank 3 which will get rejected and the realized partition is $\psi^E = \mathcal{N}$.

Next consider the Subgame in Node F. If Bank 1 has made an acceptable bid, Bank 2 is happy to stay independent and collecting $v^{\{1,3\},\{2\}}(\{2\}) = 0.11$ rather than merging with 3 and getting at most 0.005. In node C, Bank 1 anticipates that it will not get a competitive bid from Bank 2. Thus by making any acceptable bid for Bank 3, Bank 1 will sign up 3 and collect at most $v^{\{1,3\},\{2\}}(\{1,3\}) = 0.005$. It is better for Bank 1 to make an unacceptable offer to bank 3, and let the game continue to node E, where $\psi^E = \mathcal{N}$ is realized, and Bank 1 will collect $v^{\mathcal{N}}(1) = 0.01$. Thus we know that whenever the game comes to node C, the partition $\psi^C = \mathcal{N}$ will be realized.

Intuitively, if Banks 1 and 2 cannot agree to cooperate the game reaches node C and coordination failure emerges. Even though it would be welfare increasing, neither bank wants to bail out 3 because it would make them individually worse off. An inefficient outcome, where each bank stays independent is realized. Specifically Banks 1 and 2 get a payoff of $v^{\mathcal{N}}(1) = v^{\mathcal{N}}(2) = 0.01$ in the subgame starting at C.

In node B Bank 1 therefore can sign up bank 2 for any price just above 0.01. Is this worthwhile for Bank 1? When signing up 1, the game continues to node D, where Bank 1 considers signing up 3 as well. By staying independent, bank 3 can realize a value of zero. Therefore Bank 1 can sign up 3 for 0 and keep for itself $v^{\{1,2,3\}}(\{1,2,3\}) = 0.115$ minus the cost of signing up 2 and 3, that is 0.115-0.01-0=0.105.

Thus Bank 1 can realize a value of 0.105 by signing up 2 and continuing to node D, or get 0.01 by making an infeasible bid for Bank 2 and moving on to node C. Bank 1 will optimally chose the former strategy and the grand coalition will be realized. The individual banks realize payoffs of 0.105,0.01, and 0, respectively.

Intuitively Banks 1 and 2 anticipate that staying independent leads to an inefficient outcome. Acting jointly eliminates the possibility for each bank to free ride on the other and thus prevents coordination failure. This efficient equilibrium is only supported by two arrival orders: 1,2,3 and 2,1,3. These orders allow the two solvent banks to coordinate their actions before the troubled bank arrives at the bargaining site. By merging before bank 3 arrives, banks 1 and 2 can effectively reduce the three player game to a two player game, which always has an efficient solution.

Arrival order 1 3 2: Start again in node E. Banks 1 and 3 do not cooperate, bank 2 has rejected Bank 1's bid and it is now Bank 3's turn to extend an offer to Bank 2. By staying independent, Bank 2 can realize a payoff of $v^{\mathcal{N}}(\{2\}) = 0.01$. The most that 3 could bid for Bank 2 is $v^{\{\{1\},\{2,3\}\}} = 0.005$. Thus in node E, bank 2 will not bail out 3, every bank stays independent, and bank 3 fails.

In node F, Bank 1 has made a feasible bid for 2. Bank 2 will accept any bid that is at least as high as what Bank 2 gets by rejecting and ending up in node E. Thus Bank 1 has made a bid of at least 0.01. Bank 3 cannot offer more than $v^{\{\{1\},\{2,3\}\}}(\{2,3\}) = 0.005$ and therefore Bank 1 will win 2 and Bank 3 fails. In node C Bank 1 is indifferent between signing up 2 or not. By signing up 2, the game moves to node F and bank 1 will get $v^{\{\{1,2\},\{3\}\}} - 0.01 = 0.01$. By making an unacceptable bid for 2, the game continues to node E and Bank 1 collects $v^{\mathcal{N}}(\{1\}) = 0.01$. Similar to the previous case we see that when Banks 1 and 3 do not merge, coordination failure arises and an inefficient solution is realized.

To analyze node D suppose that banks 1 and 3 have merged, i.e. Bank 1 is bailing out bank 3. Then Bank 2 has a strong incentive to free ride and not contribute to the rescue of Bank 3. Formally, by staying independent Bank 2 can realize $v^{\{\{1,3\},\{2\}\}}(\{2\}) = 0.11$. Bank 2 will not accept any offer below that and when Bank 1 sign up 2 there is only $v^{\{1,2,3\}}(\{1,2,3\}) - 0.11 = 0.005$ left for banks 1 and 3 to share. Banks 1 and 3 can share the same amount if they do not sign up bank 2. At node B, bank 1 can therefore make at most 0.005 by signing up 3 which is less than the 0.01 that Bank 1 can get by making an unacceptable offer to 3 and continuing in node C. Thus, the overall outcome is that of the subgame starting in C in which each bank stays independent and bank 3 fails. The payoffs for banks 1,2, and 3 are 0.01,0.01,and 0, respectively. The solution is inefficient as the sum of the payoffs is less than what could have been achieved by the grand coalition.

The inefficiency arises whenever a solvent bank moves last. If the other solvent bank has agreed to bail out Bank 3, the last mover can stay independent and collect its full interbank payments. The other solvent bank anticipates that it will have to carry the burden of bailing out 3 alone and thus will optimally decide to stay independent. If the first solvent bank does not bail out Bank 3, the last mover has no incentive to do so either and Bank 3 fails. In this example four out of six possible arrival orders lead to an inefficient outcome. The following table summarizes the payoffs for all orders of proposers:

Order proposers	Bank 1	Bank 2	Bank 3	realized partition
1 2 3	0.105	0.01	0	$\{1,2,3\}$
1 3 2	0.01	0.01	0	$\mathcal{N}$
2 1 3	0.01	0.105	0	$\{1,2,3\}$
2 3 1	0.01	0.01	0	$\mathcal{N}$
3 1 2	0.01	0.01	0	$\mathcal{N}$
3 2 1	0.01	0.01	0	$\mathcal{N}$
Average	0.0258	0.0258	0	

The coordination problem that creditors face in some arrival orders and the resulting inefficiency can potentially resolved by regulatory intervention. Our model provides a possible explanation for recent cases like LTCM or Bear Sterns, where bank regulators intervened and coordinated the bailout negotiations.

2.5 The Failure of the Coase Theorem and an Analogy to Bank Runs

Ronald Coase, who won a Nobel prize for his work on externalities, argued that, so long as property rights are clearly established, externalities will not cause an inefficient allocation of resources. Modern writers (see, e.g., Tirole (1988)) also assert that the bargaining outcomes are ex-post efficient as long as information is perfect and transactions costs are costless. In such a world agents will write contracts that will that will enforce the efficient outcome.[10] This result in called the Coase theorem. In contrast, we find in the previous subsection that the bargaining solution is ex-post inefficient, in violation of the Coase Theorem as the positive externalities caused by the bailout of the insolvent banks by one of the solvent banks lead to a renegotiation breakdown, and avoidable liquidation costs are incurred.

In our setting, as advocated by proponents of the Coase theorem, banks enter into SIP contracts to share risks with each other. The contracts imply that stronger banks make payments to weaker banks ex-post. However, banks do not risk share perfectly since it reduces their incentives to maintain the quality of their underlying streams. Thus there is some residual unhedged risk in the financial system. The conflict arises when one bank is insolvent and two are not. In this case, while the two solvent banks may jointly agree that the insolvent bank should be bailed out, and hence its liquidation costs avoided, they disagree on *who* should bear the costs of the bailout. Each solvent bank therefore "runs" from the system by refusing to carry out the bailout since all the benefits from carrying out the bailout accrue to the other banks. Therefore, the inefficiency in the system is a coordination failure among the two solvent banks. In this sense our equilibrium, in the context of a bank threatening to withdraw its resources from the banking system, is similar to deposits withdrawing their deposits from a failing bank, and hence our equilibrium represents a "system-wide run" by solvent banks.

We also find it relevant to point out important differences of this coordination failure from the bank run problem in Diamond and Dybvig (1983). The bank run in their model is dependent on a queuing structure for deposits, and is a self-fulfilling prophecy: if all depositors do not run to the

[10] We take a quote from the economist magazine that explains the property rights argument:

> Markets find ways to take account of externalities - ways to "internalize" them, as economists say, more often than one might think. Bees are to externalities as lighthouses are to public goods. For years they served as a favorite textbook example. Bee-keepers are not rewarded for the pollination services they provide to nearby plant-growers, so they and their bees must be inefficiently few in number. Again, however, the world proved cleverer than the textbooks. Cheung (1973) studied the apple-growers of Washington state and discovered a long history of contracts between growers and beekeepers. The supposed market failure had been effectively - and privately - dealt with. (The Economist, February 17th 1996, p.67)

bank, the bank will remain solvent, and vice versa. As in their model, solvent banks run from the system rather than bail out the troubled bank. However, unlike the depositors who each behave myopically in the Diamond and Dybvig (1983) model, we explicitly allow for banks to possibly merge and have a coordinated policy of bailing out the insolvent bank. However, merging is not incentive compatible, and hence is not carried out in equilibrium.

3 Optimal Effort and SIP Contract Choices by Banks

In our model, banks must expend effort to improve the quality (mean) of their outside asset streams in Assumption 1. They also participate in the interbank market for risk sharing using the SIPs in Assumption 3. The banks have an incentive to reduce the variance of their equity value even though they are risk neutral. A reduction in risk decreases the bank's default probability and thus liquidation costs, which are borne by its equity holders ex ante. The SIPs have two components: the pure interbank loan and the swaps. Both components can potentially help share the risk, however, by effectively transferring the rights of this asset stream through the interbank transactions, reduce the incentive of banks to maintain the quality of their asset streams (the moral hazard problem).

In this section, we study the optimal choice of interbank SIP contracts in this setting, under the assumption that banks either renegotiate their SIPs ex-post and maximize their bargaining values as in Section 2, or do not renegotiate and maximize their profits obtain from the clearing payment vector determined by the regulator as in Section 1.1. We refer to the two cases as with and without renegotiations, respectively. One reason for formulating renegotiations is that it potentially mitigates the moral hazard problem, since the benefits of the effort are better captured by the bank in renegotiations, in which it is able to extract greater value from the other banks in the system.

3.1 The Banks and Social Planner's Optimization Problems

First consider the case without renegotiations. Then, we can write bank i's ex ante profit as

$$\pi^{CV}(\{i\}) = \text{Max}_{a,b,k}\left[\text{Max}_{h_i(a,b,k)}\left(E[v^{\mathcal{N}}(\{i\}) - \omega^{D,CV}(\{i\}) - \gamma \cdot h_i^2]\right)\right], \qquad (17)$$

where $v^{\mathcal{N}}(\{i\})$ is ex-post profit of bank i determined by the clearing vector shown (8), $\omega^{D,CV}(\{i\})$ is the deposit insurance premium in Assumption 2, and the expectation is taken over all realizations of the asset values, $\tilde{A}_i$ in Assumption 1. For convenience, we solve the problem in two stages, first choosing the terms of the interbank contracts (a, b, k) and then choosing the level of effort h_i

conditional on the contracts. This follows since banks asset values have identical distributions and are specified as *quid pro quo* exchanges so that the contract choices are by definition common for all firms. We solve for individual effort choices, which also turn out to be equal across banks since due to symmetry. Notice that the effort choice has an externality since the SIPs partly transfer the benefits of the improved asset stream to increasing the profits and lowering liquidation costs at other banks. Bank i can appropriate these benefits only to the extent that it obtains better recoveries when these other banks have low asset realizations. Therefore, as in any public goods problem, bank i chooses an effort level that maximizes only its personal profit, which is generally lower that the socially optimal level. The socially optimal choice of effort maximizes

$$\pi^{CV} = \text{Max}_{a,b,k} \left[\text{Max}_{h_i(a,b,k)=h(a,b,k)} \left(\sum_{i=1}^{N} E[v^{\mathcal{N}}(\{i\}) - \omega^{D,CV}(\{i\}) - \gamma \cdot h^2] \right) \right], \tag{18}$$

where the term $h_i = h$ denotes the constraint that each bank makes the same effort choice, and this level is chosen to maximize the joint profits of the banks in the system.

Similarly, we formulate the individual bank's problems with renegotiations as

$$\pi^{R}(\{i\}) = \text{Max}_{a,b,k} \left[\text{Max}_{h_i(a,b,k)} \left(E[\phi(\{i\}) - \omega^{D,R}(\{i\}) - \gamma \cdot h_i^2] \right) \right], \tag{19}$$

where $\phi(\{i\})$ is the value of bank i with renegotiations formulated in Section 2. The social planning problem is formulated analogously to (18).

Given the lack of explicit closed-form solutions for clearing vectors and bargaining values we characterize the optimal contract choices of banks with several numerical examples for the case where there are three banks. We calculate all relevant expectations with Monte-Carlo simulations. We start though with a simple analytical result for the case where banks maximize profits without renegotiating the SIP contracts ex post.

Result 4 *If banks maximize profits without renegotiations as in (17), then the choice of* $b = 0$ *and any* $a > 0$ *can never exceed the profit with* $b = 0$ *and* $a = 0$.

The result shows that pure interbank loans cannot be optimal if banks do not renegotiate their interbank claims ex post. The intuition for the result is simply that the bank's equity holders do not get any gain unless they can pay off their claims in full. However, with reciprocal pure interbank loans, they can never receive more then the face value of the amount they owe. Therefore, such loans are never optimal.

It is interesting that this result does not carry over to the case where banks renegotiate their interbank settlements. Intuition for this can be got from Lemmas 1 and 2 for the two bank case. There we showed that with renegotiations, bank i is liquidated only when $\tilde{A}_i - L_i + d(\{i\}) < 0$, otherwise the remaining bank could extract resources from bank i without it incurring liquidation costs. Since $d(\{i\}) \geq 0$, this liquidation threshold is lower than for the case when $a = 0$ and $b = 0$. Therefore, with renegotiations, pure interbank loans may be optimal.

3.2 Optimal Effort and Contract Choice

The SIPs cause externalities in effort choices through two channels. First, when the bank enters into asset swaps, it cannot capture the full benefit of its investment in effort. It will pass on $(1-(N-1)\,b)$ of the increase in the mean of $\tilde{A}$ to the other banks, while it still bears the full cost of effort. The optimal effort choice will therefore decrease in b. In contrast, interbank loans do not have this externality. This point is illustrated in the bottom panel of Figure 5, which plots the optimal effort choice for different amounts of a and b contracted. As reasoned, an individual bank's effort decreases in b but is relatively insensitive to a in this example. Second, when banks renegotiate claims ex post, both swaps and interbank loans provide some gains to the other banks in states where these banks are insolvent, and cannot pay back reciprocal amounts as contracted. Because, this channel only works when the other banks are insolvent, the externality is smaller.

The bank's profit for a given risk sharing agreement (a, b) is determined by both, the optimal effort decision given that contract and the benefit from risk sharing. The top panel of Figure 5 illustrates the bank's profit for different risk sharing agreements. The upper surface is the profit with negotiations and the lower surface for the case without negotiations. For any given choice of contract values a and b, profits are always higher with renegotiations, because the lowest payoff with renegotiations is in the case where they break down, and then the payoff of banks is as determined by the clearing vector (the payoff without renegotiations). The two surfaces join when banks under two conditions (i) When banks do not hedge (a=0, b=0), and (ii) When there is perfect risk sharing (b=1/3), since in this case the ex post values with the hedges of all banks are the same so that they are either all bankrupt or all solvent and can fulfill their SIP obligations, so no renegotiations are necessary.

One of the main results of this paper, in line with Result 4, is that pure interbank loans are are not optimal contracts even when banks renegotiate the payments on interbank claims ex post. Even though they do not adversely affect the effort choice as much as asset swaps, straight debt contracts are a poor instrument for risk sharing. Banks owe the same amount as the amounts they are scheduled to receive, and only benefit if they can renegotiate some positive net amounts while insolvent. Relative to asset swaps though, they lead to a greater incidence of insolvency. Asset swaps provide better diversification, because in states in which a bank's asset realization is low, its required payment is low as well, and thus the bank is less likely to be insolvent, and its equity holders can retain positive value. We see in Figure 5 that with or without renegotiations, banks have positive optimal quantities of asset swaps. The optimal choice is higher though for the case without renegotiations: banks choose $a = 0$ and $b = 0.07$ for this case and $a = 0.07$ and $b = 0.01$ with renegotiations. In the latter case, the bank would swap out 2% of its assets to other banks but also choose a larger amount of interbank loans. The intuition behind this is as follows: with renegotiations, a bank is able to get a larger recovery of its SIP claims if other banks default since liquidation is often avoided and therefore the harmful effects of interbank loans is mitigated. Therefore, its hedging costs (the amount it pays when it has a strong realization) are smaller.

3.3 The Impact of SIPs on Credit Risk and Systemic Risk in the Strong Bankruptcy Regime.

We now address the main questions of this paper on the effect of SIPs on credit and systemic risk. Our definition of systemic risk is the frequency of contagious defaults in (4) and (6). We study the simulated frequency of bank liquidations and expected liquidation losses for three cases: with no interbank hedging, with interbank hedging but no renegotiations, and with interbank hedging and renegotiations in Figure 6. We show two sets of bars for each case: the left bar denotes the frequency of liquidations, while the right shows the frequency of technical defaults by banks, some of which are renegotiated and do not lead to liquidations. In comparing the bars though, it is useful to note that some cases of 1 default could be followed by 2 liquidations due to systemic spillovers, and hence the left bar can in principle be higher than the right bar for a given number on the horizontal axis. It is also useful to note that without any interbank hedging the two bars are identical (top panel) as defaults always imply liquidations. It is intuitive that for a given level of effort, hedging and renegotiations should both lower credit risk and systemic risk. However, we study these effects in a setting where due to the change in optimal choices of hedging contracts and effort, the impact on both credit and systemic risk is potentially ambiguous.

For the set of parameters uses in the example above, on the one hand renegotiations reduces risk sharing, which increases the variance of the banks' payoffs and thus increases systemic risk. On the other hand, with fewer assets swapped out, banks will optimally increase their effort choice, which raises the mean of the banks payoffs and lowers systemic risk. The benchmark case, with no SIPs, in the top panels, shows that the probability of at least one liquidation is about 9%, however the probability of 3 defaults is negligible. Moving to the middle panel, the case of hedging with SIPs and no renegotiations, we find that that the probability of at least 1 liquidation declines to 4.5%, however the probability of 2 and 3 liquidations increase substantially. Looking at the rightmost bars we see that most of the cases of 3 liquidations are due to systemic (spillover) effects and not from fundamental defaults at all three banks. Therefore, the use of SIPs without renegotiations indeed seem to increase systemic risk, although they lower credit risk. Finally moving to the bottom panel, we see that the effects above get even stronger. Credit risk declines further while the incidence of systemic risk particularly for the case of 3 liquidations, increases further. About 80 percent of the cases of 1 technical default are resolved by the banks and lead to no liquidations. It is important to note that the probability of three liquidations from 3 technical default is close to zero so that all systemic concerns in arise from renegotiation breakdowns.

It is also useful to note that bank profits, as exhibited in Figure 5 are higher with renegotiations, so that social welfare (note that depositors losses are covered by deposit insurance in both settings, whose costs are subtracted from profits) is higher with higher systemic risk. The right panels show that the distribution of expected liquidation losses, which also show an increased positive skewness (correlation in liquidations) in the three cases discussed.

Since the frequency bars above carry different sets of realizations of asset values, to better clarify the role of renegotiations, in Figure 7 we look at what happens in each simulation to the cases of defaults without renegotiations, to the case with renegotiations. For example, the first block of bars shows what happens when there is one bank liquidation, without renegotiation. As seen there is a substantial benefit of renegotiations, as nearly 60 percent. However, looking at the case of 2 liquidations without renegotiations, we find a 10 percent chance of having 3 liquidations with them. This only happens because with renegotiations banks choose a lower amount of asset swaps leaving them less diversified and more susceptible to spillovers from other banks. Therefore, this figure sheds some light on the role of renegotiating SIP contracts, but still does not resolve the question of the *partial* effect of renegotiations.

To shed further light on this issue, in Table 1, we look at both credit risk (the probability of a bank being liquidated) and systemic risk (the probability of three banks being liquidated) with and without renegotiations, holding fixed the SIP contracts. In the top (bottom) panels, we use the contract that is optimally chosen for the case without (with) renegotiations. For each contract the probability of any given bank being liquidated is substantially lower with renegotiations. The difference is greater for the case of the optimal contract with renegotiations which has a smaller amount of asset swapping and hence less diversified banks. With renegotiations though, the chance of liquidation with either contract is almost equal.

3.4 The Impact of SIPs on Credit Risk and Systemic Risk in the Weak Bankruptcy Regime.

In this subsection we study the impact of SIPs on both systemic and credit risks in a weak bankruptcy regime, with and without renegotiations. We recall that the weak regime results from choosing the fixed point of the clearing vector with the smallest payments. As discussed in Example 1, this case, leads to the largest systemic risk. The results are in Figure 8, and the panels are analogous to those discussed for the strong regime. However, the difference between the panels without and with renegotiations is vast. In the case without renegotiations (middle left panel) these contagious defaults happen in a 100 percent of the simulations, since with positive liquidation costs, the domino effect of no payments spreads through all banks. However, renegotiations are mostly successful in such a setting (see Example 3) so that with renegotiations (bottom left panel), we get back an overall picture not that different from that in the strong bankruptcy regime. The weak bankruptcy regime relatively favors weaker banks ex post, but since the banks in our example our identical ex ante, has a small effect on their ex ante profits and optimal hedging and effort choices. The overall result we get is that the overall impact of the bankruptcy regime on credit risk and systemic risk are small once we allow for ex post renegotiation of financial contracts.

3.5 The Effects of Changing the Bankruptcy Cost Parameter and Asset Correlation

In this section we analyze the comparative static effects of different correlations and bankruptcy costs on bank's profits and the importance of renegotiations. For each pair of correlation and bankruptcy we compute the bank's profit for possible contracts (a,b) given an optimal effort choice. We then pick the contract that maximizes the bank's profit and record this profit as the maximum attainable profit. Figure 9 shows the maximum attainable profit for different correlations and bankruptcy costs with and without renegotiated SIP payments. When bankruptcy costs move to zero, there are no dead weight losses to the system, and hence there is no pie to renegotiate about.

Therefore renegotiations become unimportant and profits under the clearing vector and under negotiations converge. We see that a higher Φ on its own hurts bank profitability but renegotiations can mitigate the effect as banks can avoid paying the bankruptcy cost. Therefore the gap between the two surfaces increases for higher values of Φ.

We pursue this last point further by studying the comparative static effect of varying the liquidation cost parameter on total liquidation cost, which is an increasing function of both credit and systemic risks. We study the partial effect of renegotiations by holding the effort and SIP contract choices fixed. The results in Figure 10 show that liquidation costs increase less rapidly than the parameter itself. This happens, because the proportion of successful renegotiations increases in the liquidation cost parameter. As liquidation costs increase, the threat points of individual banks are lower, so that the chances of renegotiation breakdown decline.

Returning to Figure 9, we see that increased asset correlation decreases the banks profit at least when bankruptcy costs are positive. A higher correlation decreases the risk sharing opportunities, increases defaults and thus dead weight costs. Renegotiation can again mitigate the problem but become increasingly ineffective as the correlation between the assets approaches 1. In summary, from a set of ex-ante equally profitable investment opportunities, banks will chose ones with low correlation and low liquidation costs.

4 Conclusions and Extensions

We study the role of structured investment products (SIPs) in enabling banks to better hedge the risks in their asset streams but in generating greater systemic risk — the risk of financial distress spreading through the financial system — due to the linkages created by these contracts. By swapping out portions of their asset streams, banks lose the incentive to maintain the quality of their assets and compensate for lower quality with greater hedging. Banks attempt to renegotiate their SIP contracts ex post in the event of insolvencies at one or more banks to lower liquidation costs in the system. Renegotiations helps restore incentives but are unable to improve the equilibrium to the social optimum because they may break down and lead to inefficient liquidations. Breakdowns are endogenous to our model and occur in periods when several solvent banks are able to credibly threaten to 'run' from their obligations to weaker banks. The systemic transmission is greatest in periods of renegotiation breakdown. Pure interbank loans are ineffective in helping banks manage systemic risk, while asset swaps optimally chosen help increase social welfare and bank profits, lower credit risk, but increase the systemic risk of the system.

While the current version of the paper has investigated the optimal contracts with three banks, in later versions we should be able to extend the analysis to a larger number of banks. It would be interesting to characterize the renegotiation breakdown probabilities as the number of banks becomes large, approximating a perfectly competitive system. In addition, we hope to extend our results to more complex SIPs and uncover the kinds of contracts that would minimize renegotiation breakdowns, and maximize social welfare. As in recent work by Shiller (2003), such an analysis would provide guidance to the sorts of financial contracts that could increase the efficiency of the entire banking system, as opposed to individual banks.

Appendix

Proof of Result 1. For establishing the sufficiency of the conditions for a liquidation, if $\tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}} < 0$ then by cooperating and not liquidating either bank the sum of the two banks' equities is negative, while liquidation of either (or both) implies a payoff of $v^{\mathcal{F}}(\{1\}) + v^{\mathcal{F}}(\{2\}) \geq 0$, so that liquidation is an optimal policy. If $\tilde{e}_i^{\mathcal{F}} + \tilde{e}_j^{\mathcal{F}} > 0$, but (11) holds, $\tilde{e}_i^{\mathcal{F}} < 0$, and by (10) $p^{\mathcal{F}}(\{i\}) = 0$. Therefore we have $v^{\mathcal{F}}(\{i\}) = 0$ and (12) implies that $v^{\mathcal{F}}(\{j\}) = \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) > 0$. Under (11), f$\tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) > \tilde{e}_i^{\mathcal{F}} + \tilde{e}_j^{\mathcal{F}}$ so that liquidation of i gives the two players a larger amount to share relative to cooperating and canceling out all SIP contracts.

For establishing the necessity of the stated conditions suppose (11) does not hold while (12) does. Then we have $\tilde{e}_i^{\mathcal{F}} + d^{\mathcal{F}}(\{j\}) > 0$ and $\tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) > 0$. Clearly bank j can meet its SIP commitments in full. There are two subcases: (i) $0 \leq p^{\mathcal{F}}(\{i\}) < d^{\mathcal{F}}(\{i\})$, then $v^{\mathcal{F}}(\{i\}) = 0$, and and $v^{\mathcal{F}}(\{j\}) = \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) + p^{\mathcal{F}}(\{i\}) < \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) + \tilde{e}_i^{\mathcal{F}} + d^{\mathcal{F}}(\{j\}) = \tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}}$, where the last inequality obtains because (10) implies that $p^{\mathcal{F}}(\{i\}) < \tilde{e}_i^{\mathcal{F}} + d^{\mathcal{F}}(\{j\})$. (ii) If $p^{\mathcal{F}}(\{i\}) = d^{\mathcal{F}}(\{i\})$ then both banks are able to pay in full, and hence $v^{\mathcal{F}}(\{1\}) + v^{\mathcal{F}}(\{2\}) = \tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}}$. Thus cooperation is the optimal strategy for each case.

Next (11) holding and (12) not holding contradicts $\tilde{e}_i^{\mathcal{F}} + \tilde{e}_j^{\mathcal{F}} > 0$. To complete the proof of the sufficient of the conditions we need to consider the cases when $\tilde{e}_i^{\mathcal{F}} + \tilde{e}_j^{\mathcal{F}} > 0$ and neither condition holds. We then have $\tilde{e}_i^{\mathcal{F}} + d^{\mathcal{F}}(\{j\}) > 0$ and $\tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) < 0$. Again breaking up the analysis into cases we have: (i) If $0 \leq p^{\mathcal{F}}(\{i\}) < d^{\mathcal{F}}(\{i\})$, then $v^{\mathcal{F}}(\{i\}) = 0$. Then $v^{\mathcal{F}}(\{j\}) \leq \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) + p^{\mathcal{F}}(\{i\}) < \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) + \tilde{e}_i^{\mathcal{F}} + d^{\mathcal{F}}(\{j\}) = \tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}}$ as in case (i) of the previous paragraph. Note that the first inequality is strict unless $\tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) + p^{\mathcal{F}}(\{i\}) > 0$. Lastly consider the case that bank i can meet its SIP commitments in full. Then $v^{\mathcal{F}}(\{i\}) = \tilde{e}_i^{\mathcal{F}} + p^{\mathcal{F}}(\{j\}) - d^{\mathcal{F}}(\{i\})$, and $v^{\mathcal{F}}(\{j\}) \leq \tilde{e}_j^{\mathcal{F}} - p^{\mathcal{F}}(\{j\}) + d^{\mathcal{F}}(\{i\})$, with equality holding only if bank j can pay its SIPs in full. Summing the two we have $v^{\mathcal{F}}(\{i\}) + v^{\mathcal{F}}(\{j\}) < \tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}}$ so that liquidation is not optimal. ■

Proof of Result 2. Consider the case that equations (11) and (12) hold as well as $\tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}} > 0$, so that it is efficient to liquidate bank i. Suppose that bank j is the first mover and can make a take-it-or-leave-it offer to assume both the assets and liabilities of bank i as well as all its SIP commitments. Since $v^{\mathcal{F}}(\{i\}) = 0$, bank j will never make a positive bid. However, under the stated conditions, even a zero bid by bank j is not optimal, since by purchasing bank i for zero it shareholders will obtain a combined value of $\tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}} < \tilde{e}_j - d^{\mathcal{F}}(\{i\})$, the value it gets if bank i is liquidated. However, bank i will not accept a negative bid because its equity holders can obtain $v^{\mathcal{F}}(\{i\}) = 0$ by liquidating. Consider now the case where bank i gets to bid first. The lowest bid that bank j will accept is $v^{\mathcal{F}}(\{j\}) = \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\})$ and by making such a bid and merging the two banks, the shareholders of bank i will have a combined value of $\tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}} - \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) < 0$. Therefore, irrespective of the bidding order no successful bid can be made to merge the banks and bank i will be liquidated.

To prove the final statement of the result suppose both banks are able to meet their SIP commitments. Then irrespective of the bidding order neither bank will accept a bid less than $v^{\mathcal{F}}(\{i\})$. The sum of these reservation values in this case is simply $\tilde{e}_1^{\mathcal{F}} + \tilde{e}_2$, the value to be shared by merging the banks. Therefore, there are no net gains from merging irrespective of the order of proposals. ■

Proof of Result 3.

(i) We solve the game by backward induction. Start with node E. At this point, banks 1 and 2 are independent and bank 3 has rejected bank 1's bid. Thus the only player that can potentially sign up 3 is bank 2. Bank 2 bids b_2^E for bank 3 in node E. When signing up 3, bank 2 collects a payoff of $v^{\{1\},\{2,3\}}(\{2,3\}) - b_2^E$. Bank 2 is willing to bid up to the point where it is indifferent between signing up 3 and staying independent, which is given in (13).

Bank 3 can always realize a payoff of $v^{\mathcal{N}}(\{3\})$ by rejecting bank 2's bid. Thus bank 3 will accept every bid above $v^{\mathcal{N}}(\{3\})$. Since bank 2's payoff is decreasing in b_2^E, it will never bid more than bank's 3 reservation price. Thus in equilibrium we know that

$$b_2^E = \min(\bar{b}_2^E, v^{\mathcal{N}}(\{3\})) \tag{20}$$

We can now write the payoffs ϕ^E of the players conditional on reaching node E as well as the realized partition ψ^E.

(ii) The case where both 1 and 2 potentially compete for 3 is more complex. Suppose that 1 has made an acceptable bid b_1^C to sign up 3, in which case we end up in node F. Bank 2 knows that partition $\{\{1,3\},\{2\}\}$ forms when it loses the bidding war with 1. A similar logic to the case above defines the maximum $\bar{b}_2^F$ that bank 2 given in (15). Bank 2 can win the bidding war if it bids just above b_1^C, but bank 2 would never bid more than $\bar{b}_2^F$. Thus bank 2's bid is

$$b_2^F = \min(b_1^C, \bar{b}_2^F). \tag{21}$$

Next consider bank 1's position in node C: If it does not sign up bank 3, the game continues to node E, partition ψ^E will form, and bank 1 realizes a payoff of ϕ_1^E. The maximum that bank 1 is willing to bid $\bar{b}_1^C$ is where the benefit of signing up 3 minus the cost $\bar{b}_1^C$ equals the payoff of not making a bid is given by $\bar{b}_1^C$ in 14. We can then determine bank 1's optimal bidding strategy:

$$b_1^C = \min(\bar{b}_1^C, \max(v^{\mathcal{N}}(\{3\}), b_2^F). \tag{22}$$

The first term just states that bank 1 is not willing to bid more than $\bar{b}_1^C$. The second term represents the minimum bid that 1 has to make to sign up 3: By rejecting bank 1's offer, the game moves to node E and bank 3 will realize a payoff of $\phi_3^E = v^{\mathcal{N}}(\{3\}))$. Thus bank 3 will accept only bids that are above ϕ_3^E. If bank 1 wants to win, it also has to offer more than bank 2's bid of b_2^F. Substituting from Equation (21) we get:

$$\begin{aligned} b_1^C &= \min(\bar{b}_1^C, \max(v^{\mathcal{N}}(\{3\}), \min(b_1^C, \bar{b}_2^F))) = \\ &= \min(\bar{b}_1^C, \min(\max(v^{\mathcal{N}}(\{3\}), b_1^C), \max(v^{\mathcal{N}}(\{3\}), \bar{b}_2^F))), \end{aligned} \tag{23}$$

or

$$b_1^C = \begin{cases} \min(\bar{b}_1^C, \max(v^{\mathcal{N}}(\{3\}), \bar{b}_2^F)) & \text{if } b_1^C \geq v^{\mathcal{N}}(\{3\}) \\ \bar{b}_1^C & \text{if } b_1^C < v^{\mathcal{N}}(\{3\}). \end{cases} \tag{24}$$

We can now consider three cases for the solution of the subgame starting in node C:

- $\bar{b}_1^C < v^{\mathcal{N}}(\{3\})$. Bank 1 cannot offer at least as much as 3 can get for itself, bank 3 rejects bank 1's bid and the game continues to node E. The payoffs are as discussed in (i).
- $v^{\mathcal{N}}(\{3\}) < \bar{b}_1^C$ and $\bar{b}_2^F < \bar{b}_1^C$. Bank 1 can make an acceptable bid for bank 3, so the game continues to node F. Bank 1 can also afford to bid more than bank 2 and will sign up bank 3 at a price of $\max(v^{\mathcal{N}}(\{3\}), \bar{b}_2^F)$.

- $v^{\mathcal{N}}(\{3\}) < \bar{b}_1^C < \bar{b}_2^F$. Bank 2 can make an acceptable bid for bank 3 and can afford to bid more than bank 1. Bank 2 will therefore sign up bank 3 at a price just above $\bar{b}_1^C$.

(iii) Now consider the subgame at node D. Bank 2 has signed up with 1 for a bid of b_1^B and now player 1 thinks about signing up bank 3 as well. Similar to the case above, bank 1's maximum bid for bank 3 is $\bar{b}_1^D$, which makes it indifferent between merging with bank 3 or being independent, is given by (16). There is no competition for bank 3 and its reservation price is what bank 3 can get by itself which is $v^{\{1,2\},\{3\}}(\{3\})$. Bank 1 would therefore never bid more than that and bank 1's equilibrium bid is

$$b_1^D = \min(\bar{b}_1^D, v^{\{1,2\},\{3\}}(\{3\})). \tag{25}$$

(iv) We can now determine the final outcome of the game by examining node B. Bank 1 has to decide whether or not to sign up bank 2. It compares the payoff of moving to node D and collecting a payoff of $\phi_1^D(b_1^B)$ with moving to node C and collecting payoff of ϕ_1^C. It is willing to bid for 2 in node B up to the point of indifference where $\phi_1^D(\bar{b}_1^B) = \phi_1^C$, or

$$\max(v^{\{1,2\},\{3\}}(\{1,2\}) - \bar{b}_1^B, v^{\{1,2,3\}}(\{1,2,3\}) - v^{\{1,2\},\{3\}}(\{3\})) - \bar{b}_1^B) = \phi_1^C.$$

Bank 2's reservation price is what it can get by not merging with 1, thus it will accept all offers above ϕ_2^C. Bank 1 will therefore bid in equilibrium $b_1^B = \min(\bar{b}_1^B, \phi_2^C)$.

This completes the analysis of the extensive form game. ■

Proof of Result 4. To facilitate the exposition of the proof, we reverse the order of the choice of the contract terms and effort in (17), and write the profit of bank i as

$$\pi^{CV}(\{i\}) = \text{Max}_{h_i}\left[\text{Max}_{a(h_i)}\left[E\left[v^{\mathcal{N}}(\{i\}) - \omega^{D,CV}(\{i\}) - \gamma \cdot h_i^2\right]\right]\right],$$

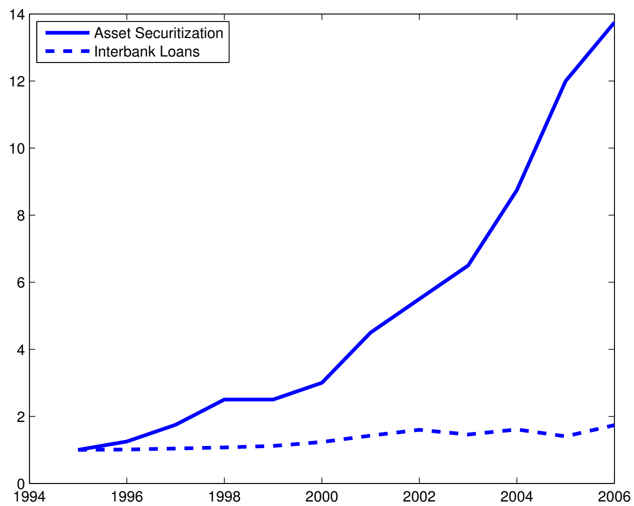
that is we first fix the effort level of the bank, and then choose the contract terms as a function of this effort level. Take any effort level h^*, fix $b = 0$ as presumed and consider the optimal choice a. With the effort choice fixed, the distributions of $\tilde{A}_i$, $i = 1, 2, 3$ are fixed. Now with a choice of any $a > 0$, the ex-post profit conditional on any realization of the asset values from the clearing vector for i is $\max(\tilde{A}_i - L_i + r^{\mathcal{N}}(\{i\}) - (N-1)\,a, 0)$, while with $a = 0$, the profit would be $\max(\tilde{A}_i - L_i, 0)$. However, by the definition of the clearing vector in (2) we have that $r^{\mathcal{N}}(\{i\}) \leq (N-1)a$, which implies that the profit with $a = 0$ is greater than or equal to the profit with $a > 0$. This completes the proof. ■

Table 1: Credit Risk and Systemic Risk With or Without Ex Post Renegotiations of Optimal SIP Contracts in the Strong Bankruptcy Regime

		Without Renegotiations	With Renegotiations
Optimal Contract without Renegotiations	Bank Liquidation Probability	2.77%	1.48%
	Expected Loss	0.0154	0.0073
	Probability 3 Banks Liquidated	0.6%	0.6%
	Effort	0.149	0.149
	Successful renegotiations		58.7% of bankruptcies
Optimal Contract with Renegotiations	Bank Liquidation Probability	3.68%	1.38%
	Expected Loss	0.0210	0.0078
	Probability 3 Banks Liquidated	0.85%	0.85%
	Effort	0.165	0.165
	Successful renegotiations		85.5% of bankruptcies

“ Bank Liquidation Probability” is the probability that a single bank will be liquidated, “Expected Loss” is the expected liquidation cost, and “Probability 3 Banks Liquidated” is the probability that all three banks will default, a measure of systemic risk. For the results in first the first three lines we use the optimal contract for a profit maximizing bank without renegotiations ($a = 0.00$, $b = 0.06$). For lines four to six we use the optimal contract for the profit maximizing bank with renegotiations ($a = 0.07$, $b = 0.01$). The other parameters are $\mu_0 = 0.3, \mu_1 = 0.42, \gamma = 1.8, \rho = 0.3, \sigma = 0.2, \phi = 0.6$.

Figure 1:



The interbank loans outstanding series is obtained from the "Assets and Liabilities of Commercial Banks in the United States" tables, H.8 (510) – item 13, from the Federal Reserve Board. The securitized bank series is from the International Monetary Fund and is the sum of all Collateralized Debt Obligations (CDO), Asset-Backed Securities (ABS), and Mortgage-Backed Securities (MBS) held at commercial banks. Both initial values have been rescaled to 1 to illustrate the difference in growth rates of the two series.

Figure 2: Structure of Banking System with Interbank SIP Hedges

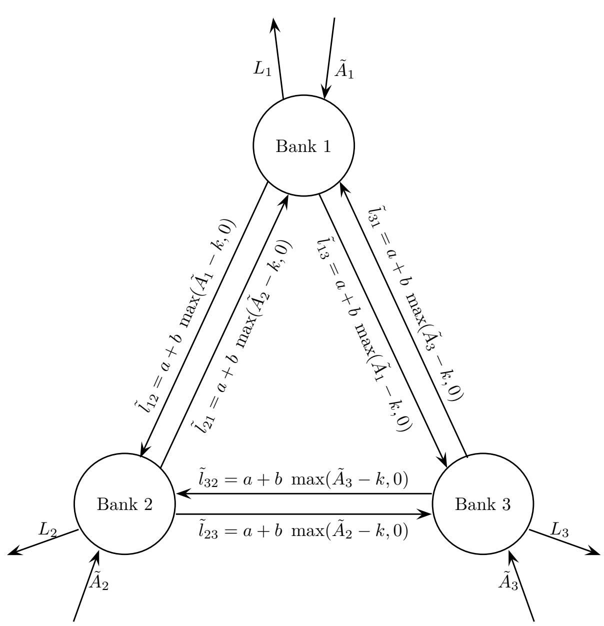
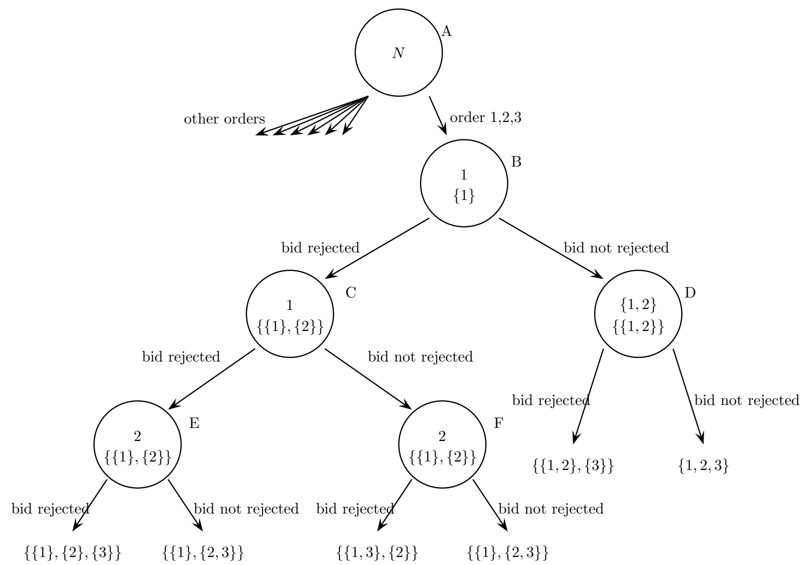


Figure 3: Extensive form of the three player game



Each node has the active player (first line) and the partition, which is realized (second line). First, at node A, Nature chooses an order in which players arrive at the bargaining site. The figure illustrates the game for the natural order 1,2,3. At node B, bank 1 makes a bid for bank 2. If the bid is not rejected, banks 1 and 2 merge (node D) and the merged bank can then make a bid for bank 3. If bank 3 accepts this bid, the grand coalition of banks forms, otherwise it remains independent. If bank 1's bid for bank 2 is rejected, then the two banks remain independent and are both potential acquirers of bank 3 (node C). Bank 1 bids first and if its bid is rejected by bank 3 the game moves to node E where bank 2 can make a bid for bank 3. If bank 1's bid is not rejected by bank 3, then the game moves to node F where bank 2 can make an additional bid for bank 3. Bank 3 chooses the higher of the bids.

Figure 4: Example to Illustrate Inefficient Liquidation.

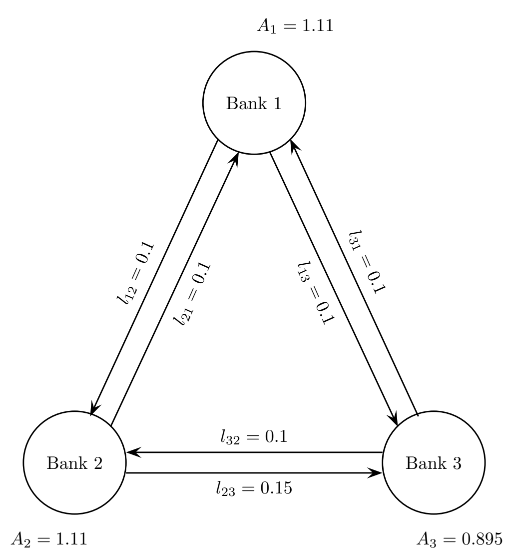
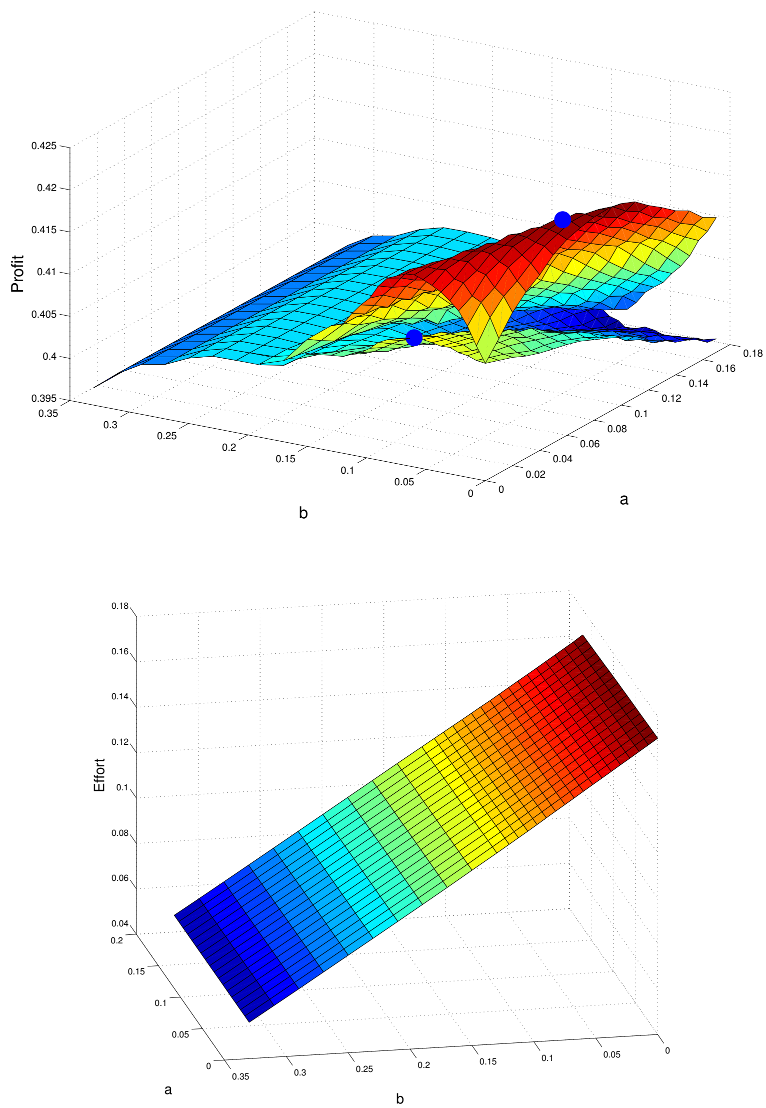
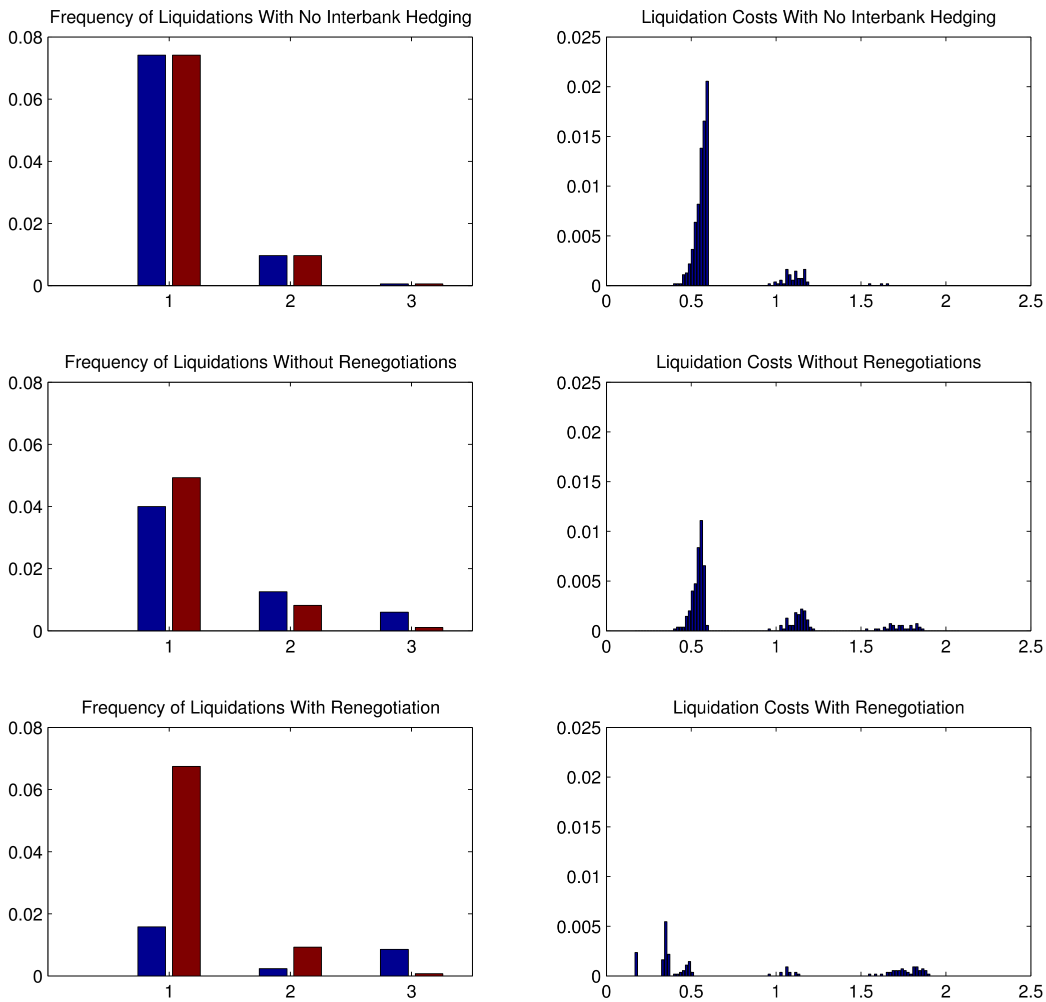


Figure 5: Bank Profits and Effort Choices for Alternative SIP Contracts



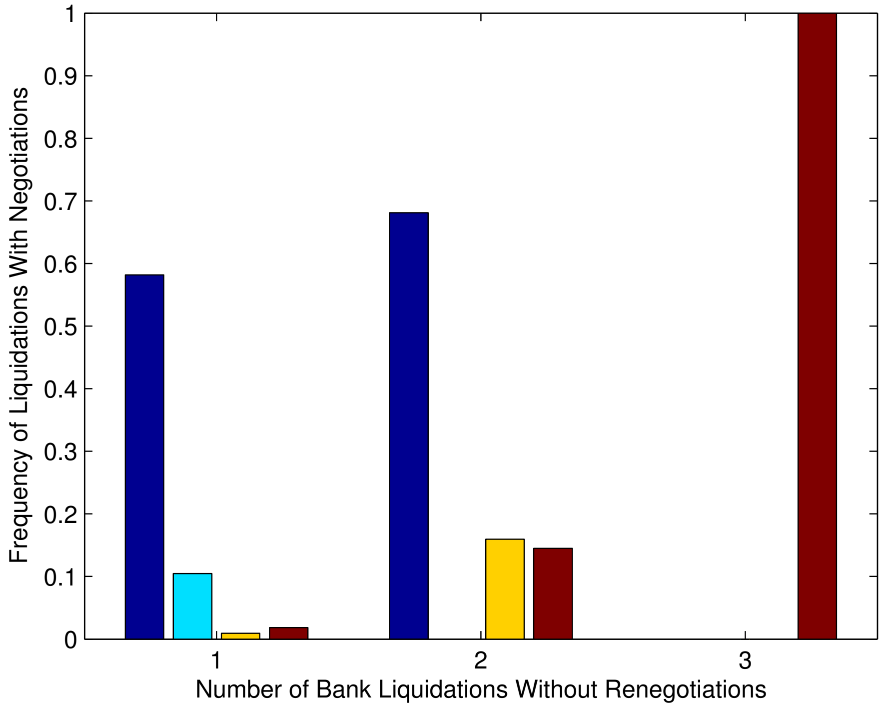
We display the profit (top panel) and effort choices (bottom panel) of the individual bank for different exposures of straight debt a and asset swaps b. The upper surface is with negotiations, the lower surface for the case without negotiations. The parameters are: $\mu_0 = 0.3, \mu_1 = 0.42, \gamma = 1.8, \rho = 0.3, \sigma = 0.2, \phi = 0.6$.

Figure 6: Distribution of Default Frequency and Expected Loss with SIPs in the Strong Bankruptcy Regime



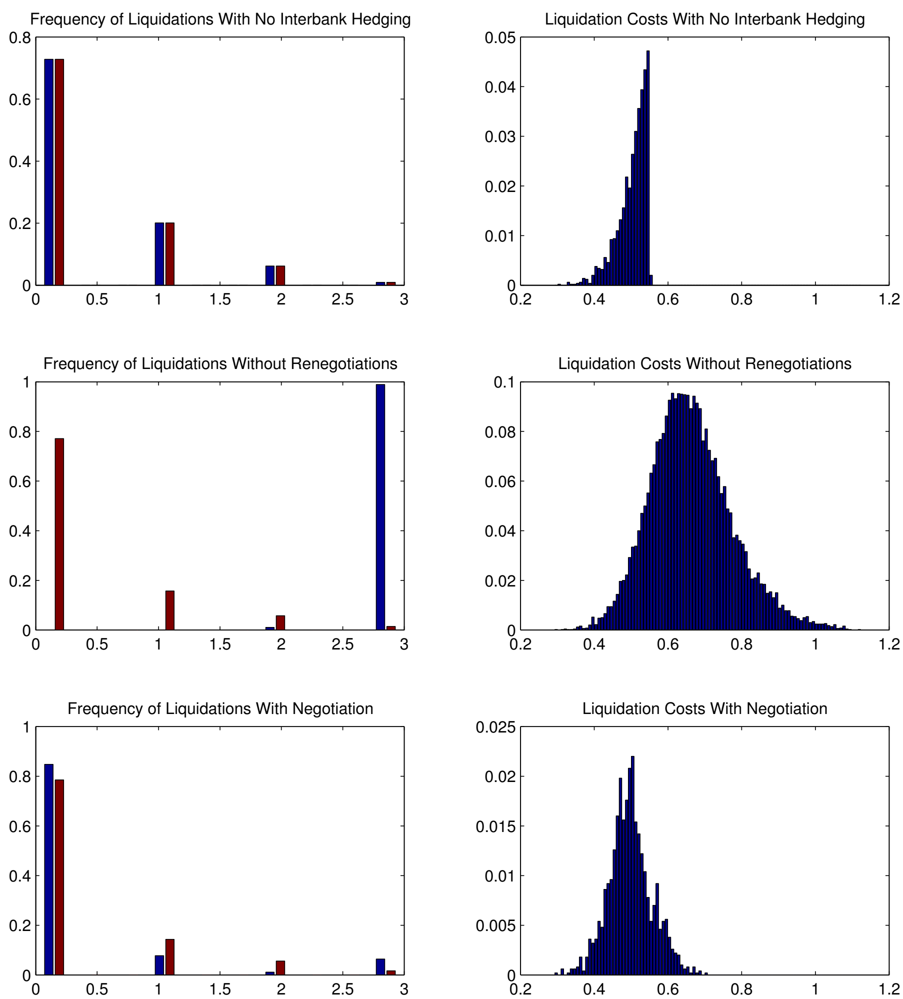
In the **left panels**, we plot two bars at each integer 1,2,3, the possible number of liqudiations or fundamental defaults that will be observed in a three bank system. The left and right bars display the frequency of liqudiations and fundamental defaults, respectively. We present the simulated frequency for three cases: with no interbank hedging (**top panels**), with interbank hedging but no renegotiations (**middle panels**), and with interbank hedging and renegotiations (**bottom panels**). The **right panels** shows histograms of dead weight losses given at least one default for the three cases. The parameter values used for the results are: $\mu_0 = 0.3, \mu_1 = 0.42, \gamma = 1.8, \rho = 0.3, \sigma = 0.2, \phi = 0.6$.

Figure 7: What happens to Liquidations Without Renegotiations when Renegotiations are Allowed in The Strong Regime



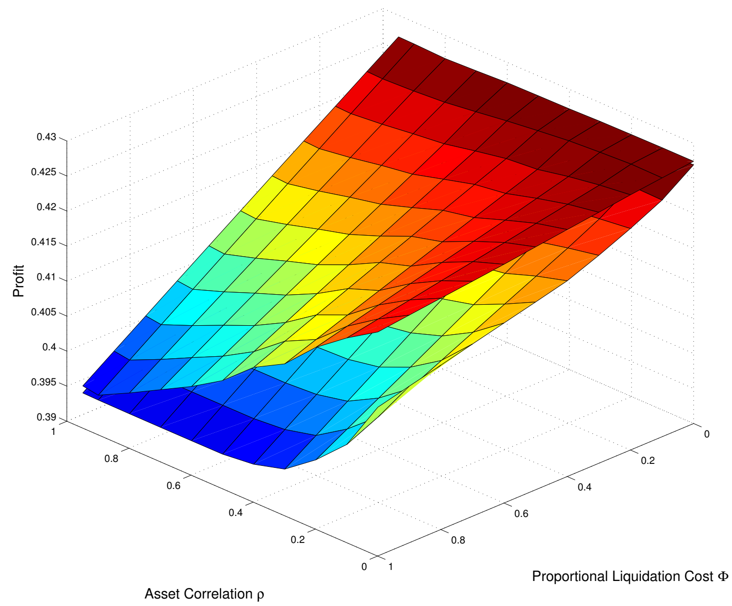
The parameter values used for the results are: $\mu_0 = 0.3, \mu_1 = 0.42, \gamma = 1.8, \rho = 0.3, \sigma = 0.2, \phi = 0.6$

Figure 8: Distribution of Default Frequency and Expected Loss with SIPs in the Weak Bankruptcy Regime



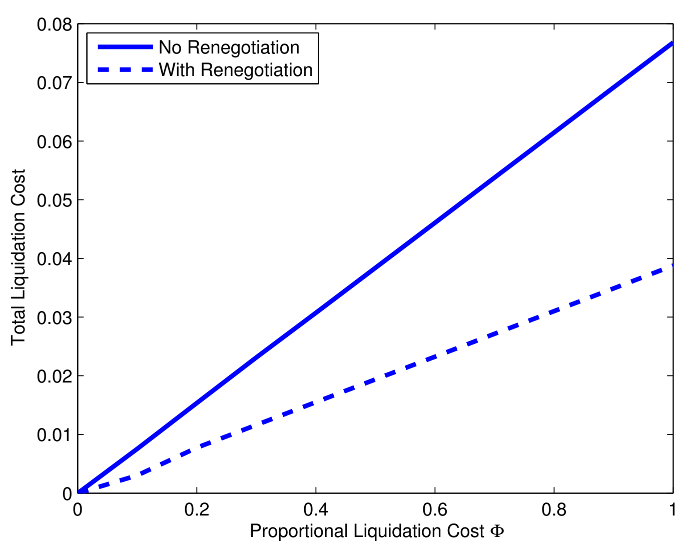
In the **left panels**, we plot two bars at each integer 1,2,3, the possible number of liqudiations or fundamental defaults that will be observed in a three bank system. The left and right bars display the frequency of liqudiations and fundamental defaults, respectively. We present the simulated frequency for three cases: with no interbank hedging (**top panels**), with interbank hedging but no renegotiations (**middle panels**), and with interbank hedging and renegotiations (**bottom panels**). The **right panels** shows histograms of dead weight losses given at least one default for the three cases. The parameter values used for the results are: $\mu_0 = 0.3, \mu_1 = 0.42, \gamma = 1.8, \rho = 0.3, \sigma = 0.2, \phi = 0.6$.

Figure 9: Optimal Contract Choice as Functions of Asset Correlation and Proportional Bankruptcy Costs



Profit of the individual bank for different correlations ρ and bankruptcy costs ϕ given the banks optimal contract choice. The parameters are: $\mu_0 = 0.3, \mu_1 = 0.42, \gamma = 1.8, \sigma = 0.2, \phi = 0.6$.

Figure 10: Liquidation Costs as a Function of the Liquidation Cost Parameter Φ



The parameters are: $\mu_0 = 0.3, \mu_1 = 0.42, \gamma = 1.8$, $\rho = 0.3$, and $\sigma = 0.2$ The contract parameters at $a = 0$, $b = 0.06$ and $h = 0.15$ are held constant, which are the optimal contract choices for the case $\Phi = 0.6$.

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