Is Idiosyncratic Volatility Risk Priced? Evidence from the Physical and Risk-Neutral Distributions *

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Abstract

We use simultaneous data from equity, index and option markets in order to estimate a single factor market model in which idiosyncratic volatility is allowed to be priced. We model the index dynamics' P-distribution as a mean-reverting stochastic volatility model as in Heston (1993), and the equity returns as single factor models with stochastic idiosyncratic volatility terms. We derive theoretically the underlying assets' Q-distributions and estimate the parameters of both P- and Q-distributions using a joint likelihood function. We document the existence of a common factor structure in option implied idiosyncratic variances. We show that the average idiosyncratic variance, which proxies for the common factor, is priced in the cross section of equity returns, and that it reduces the pricing error when added to the Fama-French model. We find that the idiosyncratic volatilities differ under P- and Q-measures, and we estimate the price of this idiosyncratic volatility risk, which turns out to be always significantly different from zero for all the stocks in our sample. Further, we show that the idiosyncratic volatility risk premiums are not explained by the usually equity risk factors. Finally, we explore the implications of our results for the estimates of the conditional equity betas.

Keywords: Idiosyncratic volatility risk premium; joint estimation; option return; factor models.

JEL Classification: G10; G12; G13.

1 Introduction

The most important result of the capital asset pricing model (CAPM) states that only the systematic risk is priced in equilibrium and idiosyncratic risk is not. Some earlier studies such as Levy (1978), Merton (1987), and Xu and Malkiel (2003) challenged this finding, by suggesting that investors may not be able to diversify properly. In such a case idiosyncratic risk should be positively related to the expected stock returns to compensate for this imperfect diversification. Although the definition of idiosyncratic risk has changed over time because of various redefinitions of the CAPM[1], the very definition of idiosyncratic risk implies that it should be uncorrelated or, if the non-diversification argument is accepted, positively correlated to expected stock returns.

In the light of this theory, the results of the influential study of Ang et al. (2006), constitute a puzzle, since they document the underperformance of the stocks with high idiosyncratic return volatilities. Several subsequent studies have tried to explain this puzzle. Chen and Petkova (2012) propose that idiosyncratic volatility is priced because it correlates with changes in the average equity return variance, which is part of the aggregate variance. Duarte et al. (2012) introduce the predictive idiosyncratic variance component that correlates with macro economic factors, and argue that the puzzling findings of Ang et al. (2006) do not hold when portfolios are sorted based on the unpredicted idiosyncratic volatilities. Other papers suggested that the puzzling findings are due to the choice of frequency, weighting, illiquidity, or the specific measurement of volatility.[2]

Another set of relevant CAPM studies examined the issues of beta estimation and systematic risk through the option market. Buss and Vilkov (2012, BV) and Chang et al. (2011, CCJV) pointed out that information extracted from that market is forward-looking, while the usual CAPM estimations rely on historical data. Hence, CAPM results extracted from the option market may be better proxies for future estimates of systematic risk than those stemming from the conventional approach. Christoffersen et al. (2013, CFJ) adopt a similar reasoning and estimate betas from a cross section of index and equity options, assuming that the idiosyncratic volatility is not priced.

A key issue in using the option market for the CAPM estimations is the change in probability measure between underlying and option markets. Once the complete markets assumption is abandoned, the underlying asset return distribution extracted from the option market, the risk neutral or Q-distribution, differs from the one observed in the underlying asset market, the physical or P-distribution. In particular, the P-distribution market volatility differs from the risk neutral volatility by the price of volatility risk. The CAPM studies relying on option market data have addressed the issue by correcting the volatility through an ad hoc modeling of the correlation matrix of the returns as in BV or by adopting several assumptions about the structure of idiosyncratic risk as in CCJV. All these studies rely only on option market data for their empirical work and assume explicitly that idiosyncratic risk is not priced, or that idiosyncratic volatility is the same under both P- and Q-distributions.

[1] In particular, the conditional CAPM and the Fama-French (1993) model.

[2] See Huang et al. (2010), Bali and Cakici (2008), Han and Lesmond (2011), and Fu (2009).

In this paper we combine the two strands of literature, by investigating the pricing of idiosyncratic risk using option-implied volatilities. We estimate the parameters of the P- and Q-distributions from both underlying and option market data, and we assume in our underlying market model that idiosyncratic volatility is priced. Our empirical results reject decisively the hypothesis that idiosyncratic risk is not priced.[3] To the best of our knowledge this is the first study that links the physical and risk neutral distributions of idiosyncratic volatilities.

Our estimation methodology has also implications for equity betas, which are very important in portfolio selection and corporate finance. A popular estimation method is to use a rolling window of historical returns. Several studies have proposed using option prices to obtain forward looking estimates of stock betas. In our modeling framework beta enters the equity price dynamics and is estimated directly as part of the structural parameters. Moreover, we estimate the stock beta using the information in both returns and options prices, taking into account the market variance and idiosyncratic variance risk premiums. Further, we develop a new procedure to estimate conditional equity betas, which we estimate out-of-sample, using only the option prices observed on a given day.

We use a continuous-time modeling framework that allows for a factor structure in equity returns, where the factor is the market return. The idiosyncratic return volatility of the stock ($IVol$) follows a square-root stochastic process and is allowed to be priced.[4] We estimate the model parameters and idiosyncratic volatility state variables, conditional on market parameters and state variables, using the joint information from the equity option prices and equity returns. Our data set contains historical returns and option prices for the market index and 27 blue chip stocks over the period 1996 to 2011, and contains more than 3.4 million option quotes. Our estimation is based on a likelihood function that has a return component and an option component, while the structural parameters are internally consistent between the P and Q measures. The simultaneous estimation of the P- and Q-distribution parameters allows us to filter the spot idiosyncratic volatilities under both distributions, using an internally consistent set of parameters.

Previous studies document a strong factor structure in implied volatility levels, moneyness slopes and term structure slopes, with the factor being the market implied volatility. In our results we find that even after removing the market return factor from the equity returns there is still a strong factor structure left in the implied idiosyncratic volatility ($IIVol$) levels, slopes and term structure slopes, very similar to that observed in total implied volatilities. The first two principal components of the $IIVol$ levels explain 58% and 23% of the cross-sectional variations, respectively. The first common component has a 99% correlation with the average implied idiosyncratic volatility levels of all firms. Further, the first and the second common components have a correlation of 65% and 55% with the index implied volatility levels, respectively. The first two principal components of $IIVol$ moneyness slopes explain

[3]Note that our idiosyncratic volatility is not the same as in the study of Ang et al. (2006), since it is extracted from a single factor model and not from the Fama-French (1993) factors. The latter cannot generate an option pricing model.

[4]This modeling framework was also used by CFJ, except that these authors assumed that $IVol$ is the same under the P- and Q-distributions.

48% and 6% of the cross-sectional variations, respectively. The first common component has a 99% correlation with the average implied idiosyncratic volatility slopes of all firms. The first and the second common components have a correlation of 42% and 8% with the index implied volatility slope, respectively. Finally, the first two principal components of the term structure slopes explain 61% and 7% of the variations, respectively. The first principal component has a 99% correlation with the average implied idiosyncratic volatility term structure slopes of all firms. The first and the second common components have a correlation of 78% and -13% with the index implied volatility term structure slope, respectively.

These findings are consistent with those of Herskovic et al. (2013), who show that there is a strong factor structure in idiosyncratic volatilities, similar to that in total volatilities, by looking at firms fundamentals. Our findings complement the empirical literature that shows there are common factors in the idiosyncratic volatilities of stock returns under the P distribution, by documenting the existence of the same factor structure in distributions extracted from equity options prices.

We use the average idiosyncratic variance, AIV, as the potentially priced risk factor, and we test whether this factor can help explain the cross section of equity returns. We show that the AIV factor can reduce the pricing error of the Fama-French 25 size and value portfolios. Moreover, AIV has a positive risk premium, and its cross sectional explanatory power, in our sample period, is more than that of the HML and SMB factors.

Further, we derive the expected option return, and we form portfolios that contain the equity option, the stock, the index option, and the market index. These portfolios are formed and rebalanced in such a way that they are only exposed to the idiosyncratic variance of the equity. The return on these portfolios can be considered as the risk premium for the equity idiosyncratic variance. Using calls and puts with different moneyness ratios, we present evidence of the existence of the idiosyncratic variance risk premium.

Our estimation results show that the idiosyncratic volatility is priced and it can bear a negative or positive sign. Moreover, we define a measure of the idiosyncratic variance risk premium, defined over a 30-day period as the difference between the expected integrated idiosyncratic variance under the P and Q measures that is only partially driven by the market volatility risk premium. Further, the idiosyncratic volatility risk premium is significantly different from zero for all of the stocks in our sample, and has different signs for different stocks.

We show that the market return and the Fama-French factors, as well as the momentum factor cannot explain the time-series variations in the idiosyncratic volatility risk premiums. These time series variations can, however, be partly explained by the market variance risk premium. Further, we show that the average variance risk premium of all firms, $AIVRP$, together with the component of market variance risk premium orthogonal to $AIVRP$, have a strong explanatory power in the time-series variations of the idiosyncratic variance risk premiums.

The rest of the paper is organized as follows. In section 2 we present the model. Section 3 contains the description of the data and the estimation methodology, as well as the results regarding the common structure in idiosyncratic volatilities. In section 4 we discuss the ability of our proposed factor in explaining the cross section of equity returns. Section 5 presents the measure and the properties of the idiosyncratic volatility risk premiums. In section 6 we discuss the estimation of conditional betas. Section 7 cocludes.

2 The Model

Here we present an equity option valuation model using a single-factor structure that links the equity return dynamics to the market return dynamics. We model an equity market consisting of N stocks and a market index. The individual stock prices are denoted by $S_{i,t}$ for $i = 1, 2, \ldots, N$, and the market index price is denoted by S_t. We assume that investors have access to a risk free bond whose return is r.

We assume the following stochastic volatility dynamics for the market index under the physical distributions (hereafter P):

$$\begin{aligned} dS_t/S_t &= (\mu)dt + \sqrt{v_t}dz_t, \\ dv_t &= \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}dw_t \end{aligned} \tag{2.1}$$

As in Heston (1993), θ is the unconditional average variance, κ captures the speed of mean reversion of v_t to its long-run average, and σ measures the volatility of variance. The market equity risk premium is represented by μ. The correlation between the shocks to market return and its variance is represented by ρ, and it captures the market leverage effect.

Further, we assume that the stock return follows a one-factor model, where the factor is the excess return on the market. The volatility of the idiosyncratic part of the stock return, referred to as idiosyncratic volatility ($IVol$), is assumed to be stochastic, and to follow a square-root type process. The following describes the stock price dynamics for firm i:

$$\begin{aligned} dS_{i,t}/S_{i,t} &= (\mu_i)dt + \beta_i(dS_t/S_t - rdt) + \sqrt{\xi_{i,t}}dz_{i,t}, \\ d\xi_{i,t} &= \kappa_i(\theta_i - \xi_{i,t})dt + \sigma_i\sqrt{\xi_{i,t}}dw_{i,t} \end{aligned} \tag{2.2}$$

where, $dS/S - rdt$ is the instantaneous excess return on the market, β_i is the market beta, μ_i is the idiosyncratic return, ξ_i is the variance of the idiosyncratic return[5] , σ_i is the volatility of the idiosyncratic variance, κ_i is the speed of mean reversion for idiosyncratic volatility, θ_i is the long-run average of the idiosyncratic volatility, and ρ_i is the correlation between the shocks to idiosyncratic return and its variance.

Proposition 1. *The market index has the following dynamics under the risk-neutral measure (hereafter Q):*

$$\begin{aligned} dS_t/S_t &= rdt + \sqrt{v_t}d\tilde{z}_t, \\ dv_t &= \tilde{\kappa}(\tilde{\theta} - v_t)dt + \sigma\sqrt{v_t}d\tilde{w}_t \end{aligned} \tag{2.3}$$

[5] Here the idiosyncratic return is defined as the excess stock return in a one-factor model framework.

where, $\tilde{\kappa} = \kappa + \lambda$, and $\tilde{\theta} = \frac{\kappa\theta}{\kappa+\lambda}$, and where λ is the price of market volatility risk as in Heston (1993). Moreover, the equity dynamics under Q is as follows

$$\begin{aligned} dS_{i,t}/S_{i,t} &= rdt + \beta_i(dS_t/S_t - rdt) + \sqrt{\xi_{i,t}}d\tilde{z}_{i,t}, \\ d\xi_{i,t} &= \tilde{\kappa}_i(\tilde{\theta}_i - \xi_{i,t})dt + \sigma_i\sqrt{\xi_{i,t}}d\tilde{w}_{i,t} \end{aligned} \tag{2.4}$$

where, $\tilde{\kappa}_i = \kappa_i + \lambda_i$, and $\tilde{\theta}_i = \frac{\kappa_i\theta_i}{\kappa_i+\lambda_i}$, and where λ_i is the price of idiosyncratic volatility risk.

Proof. See Appendix A. □

The above model has been used by Christoffersen et al. (2013) who, however, assume that the idiosyncratic volatility is not priced and that the idiosyncratic variance follows the same dynamics under the P and Q distributions. They discuss the consistency of their model with some of the empirical evidence in the equity option literature such as Duan and Wei (2009) and Dennis and Mayhew (2002). Moreover, they derive a close form solution for the equity option price, and present estimation results based on equity options.

The assumption that the idiosyncratic volatility is not priced, is equivalent to implying that the market excess return is the only priced factor. There is, however, significant evidence that there are other priced factors in the economy. If the CAPM is miss-specified and there are other priced factors, then the idiosyncratic variance would consist of exposure to those missing factors, and the price of the idiosyncratic variance would reflect the linear combination of the prices of the variance of the missing factors. This is a testable hypothesis, which we test by relaxing the assumption of non-priced $IVol$, and letting the idiosyncratic volatility dynamics be different under the P and Q distributions.

Following Heston (1993), we assume that the price of the $IVol$ risk is proportional to the level of the idiosyncratic variance. Based on this assumption, the same closed form solution for the European equity options that Christoffersen et al. (2013) derive holds in our framework.[6] In our empirical work we test the hypothesis of priced idiosyncratic volatility by using information from both equity returns and equity options and verifying whether the idiosyncratic volatility dynamics are different under the physical and risk-neutral distributions.

3 Estimation and Results

There are several approaches to estimating stochastic volatility models. The main challenge in estimating stochastic volatility models is the filtering of the unobserved volatility. One approach is to treat the unobserved volatility as a parameter, and estimate all parameters using a single cross section of option prices. This is done by Bakshi et al. (1997). Another approach is to use multiple cross sections of option prices. However, for every cross section, a different initial volatility estimate is required. Bates (2000) and Huang and Wu (2004) use this approach. A third approach provides a likelihood-based estimation that can combine

[6] For the derivation of the option price formula please refer to Christoffersen et al. (2013).

the information form the option data and the underlying returns, and imposes consistency between the P and Q distributions. Ait-Sahalia and Kimmel (2007), Eraker (2004), Jones (2003), and Bates (2006) provide an MCMC analysis within this framework. A last group of papers takes a frequentist approach that can also combine the information from the option prices and the underlying returns. Chernov and Ghysels (2000) use the efficient method of moments, while Pan (2002) uses a method of moments technique. Santa-Clara and Yan (2010) and Christoffersen et al. (2013) use likelihood functions that contain a return component and an option component. Our empirical setup is most closely related to this last group of papers.

3.1 Data

We collect option data for the S&P 500 and for 27 equities, all components of the Dow Jones index. We did not include in our sample Bank of America, Kraft Food, and Travellers because of data unavailability. The option data that we use comes from the OptionMetrics volatility surface, which is based on the bid-ask midpoint. Our data spans the period from January 4, 1996, to December 29, 2011. We focus on options with maturity of up to six months. Since our estimation is computationally very demanding, we excluded options with longer maturities to keep the estimation manageable. Moreover, our data contains out-of-the-money options with moneyness[7] less than 1.1 for calls and greater than 0.9 for puts. We filter out options with implied volatility less than 5% and greater than 150%, and options that violate the apparent arbitrage conditions as described in Bakshi et al. (2003).

We also collect data for the index levels, daily returns, and dividend yield, as well as stock prices, returns, and cash dividends from CRSP. The implied volatility surface data is calculated using binomial trees. When evaluating the options model price, we subtract for every option on every stock on every day the present value of dividends, assumed to be known during the life of the option, from the stock price on that day, and we treat the option as European. The discounting is done using the appropriate interest rate estimated by linear interpolation of the Zero Coupon Yield Curve available from OptionMetrics. We do the same discounting for the index using the index dividend yield.

Table 1 presents the names of the companies in our sample as well as the number of calls, puts, and total options for each firm, and for the market index. The number of option contracts is highest for the S&P500 index. On average there are 120,811 options available for each firm, with Cisco and Johnson & Johnson having the lowest and highest number of contracts among all firms, respectively. Our estimation is based on a total of 3,430,176 option quotes for the market and all equities.

In Tables 2 and 3 we report the sample average implied volatility, minimum and maximum implied volatilities, along with average option delta, option vega[8], and average days-to-maturity of all calls and puts, separately. The average implied volatility of the market in our sample is 19.2%. Cisco and Johnson & Johnson have the maximum and minimum average

[7] Moneyness is defined as the strike price divided by the underlying asset's price.

[8] These are Black-Scholes vegas evaluated at the implied volatilities.

implied volatility in our sample. Moreover, the average days-to-maturity is close to 80 days for all firms and the market index.

3.2 Joint Estimation

In order to capture the difference between the physical and risk neutral distributions of the equity idiosyncratic volatilities, it is required to fit both distributions using the same internally consistent set of structural parameters. We do so by using a joint likelihood function that has two components, one based on returns and one based on options. Since the market variance and equity idiosyncratic variance are unobserved, we filter these sate variables by using the Particle Filter (PF) method. The PF methodology offers a convenient filter for nonlinear models such as the stochastic volatility models and is used extensively in engineering, with some applications in finance.[9]

Our estimation consists of two steps. First, we estimate the market's parameters and the filtered spot market variances. Then conditional on the market model's parameters and the spot market variances, we estimate each equity's parameters and the spot idiosyncratic variances. In what follows we describe the detailed estimation procedure.

3.2.1 Market Model

Here we describe the estimation of the market model, presented in (2.1) and (2.3). We describe how the return-based and the option-based likelihood functions are calculated, and finally how the parameters and the spot variances are estimated. Applying Ito's lemma to (2.1) we get:

$$\begin{aligned} &dln(S_t) = (\mu - \frac{1}{2}v_t)dt + \sqrt{v_t}dz_t, \\ &dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}dw_t \end{aligned} \tag{3.1}$$

The above equation shows how the unobserved volatility states are related to the observed index prices. This relationship allows the filtering of the market spot volatilities form the returns. First we discretize the model in (3.1). We use Euler scheme to get:

$$\begin{aligned} &ln(S_{t+\Delta t}) - ln(S_t) = (\mu - \frac{1}{2}v_t)\Delta t + \sqrt{v_t}z_{t+\Delta t}, \\ &v_{t+\Delta t} = v_t + \kappa(\theta - v_t)\Delta t + \sigma\sqrt{v_t}w_{t+\Delta t} \end{aligned} \tag{3.2}$$

where, $z_{t+\Delta t}$ and $w_{t+\Delta t}$ are normal random variables with mean zero and variance Δt. We implement the discretized model in (3.2) using daily index log-returns, but all the results are expressed in annual terms.

[9]For other studies that use the PF method see Christoffersen et al. (2010), Johannes et al. (2009) and Gordon et al. (1993).

The PF method approximates the true density of the variance state $v_{t+\Delta t}$ by a set of "particles" that are updated through the equations in (3.2). At any time $t+\Delta t$ we generate N particles $\{v^j_{t+\Delta t}\}_{j=1}^N$ from the empirical distribution of $v_{t+\Delta t}$, conditional on N particles $\{v^j_t\}_{j=1}^N$ from the empirical distributions of v_t. This particular implementation of the PF is referred to as the sampling-importance-resampling (SIR) PF and follows Pitt (2002).[10]

Starting from a vector of particles $v^j_1 = \theta \, \forall j$, on every day t we simulate a new set of parcels,$\{\tilde{v}^j_{t+\Delta t}\}$ from the set of smoothly resampled particles $\{v^j_t\}$, according to (3.2), as follows:

$$\begin{aligned} z^j_{t+\Delta t} &= (ln(S_{t+\Delta t}/S_t) - (\mu - \frac{1}{2}v^j_t))/\sqrt{v^j_t} \\ w^j_{t+\Delta t} &= \rho z^j_{t+\Delta t} + \sqrt{1-\rho^2}\epsilon^j_{t+\Delta t} \end{aligned} \tag{3.3}$$

where $\epsilon^j_{t+\Delta t}$ are independent normal random variables with mean zero and variance Δt. Replacing (3.3) into the variance dynamics in (3.2), we get a simulated set of particles.

$$\tilde{v}^j_{t+\Delta t} = v^j_t + \kappa(\theta - v^j_t)\Delta t + \sigma\sqrt{v^j_t}w^j_{t+\Delta t} \tag{3.4}$$

So far we have N possible values for $v_{t+\Delta t}$. Now we want to give weights to the simulated particles. The weight for every particle, $\tilde{W}^j_{t+\Delta t}$, is the likelihood that the next day return at $t+2\Delta t$ is generated by this particle.

$$\tilde{W}^j_{t+\Delta t} = \frac{1}{\sqrt{2\pi\tilde{v}^j_{t+\Delta t}\Delta t}} \cdot exp\Bigg(- \frac{1}{2}\frac{\Big(ln(\frac{S_{t+2\Delta t}}{S_{t+\Delta t}}) - (\mu - \frac{1}{2}\tilde{v}^j_{t+\Delta t})\Delta t\Big)^2}{\tilde{v}^j_{t+\Delta t}\Delta t}\Bigg) \tag{3.5}$$

We can then get the probability of each particle by normalizing the weights:

$$\breve{W}^j_{t+\Delta t} = \frac{\tilde{W}^j_{t+\Delta t}}{\sum_{j=1}^N \tilde{W}^j_{t+\Delta t}} \tag{3.6}$$

At this point we have the set of the raw particles and the associated probabilities, from which we can apply Pitt (2002) algorithm to get the empirical distribution of smoothly resampled particles. These particles can be used to simulate the next period particles until we have the empirical distributions of variances for each day in the sample.

The return-based likelihood function, which is a function of the P-distribution parameters $\Theta \equiv \{\mu, \kappa, \theta, \sigma, \rho\}$, can be defined as follows:

[10] We refer to Christoffersen et al. (2010) and Pitt (2002) for a more detailed description of the PF algorithm.

$$\ln L^R \propto \sum_{t=1}^{T} ln\Big(\frac{1}{N}\sum_{j=1}^{N}\tilde{W}_t^j(\Theta)\Big) \tag{3.7}$$

The P-measure filtered spot variance v_t^P would be the average of the smoothly resampled particles.

$$\hat{v}_t^P = \frac{1}{N}\sum_{j}^{N} v_t^j \tag{3.8}$$

Moreover, the filtered shocks to the index return, conditional on the filtered spot variance would be:

$$\hat{z}_{t+\Delta t}^P = (ln(S_{t+\Delta t}/S_t) - (\mu - \frac{1}{2}\hat{v}_t^P))/\sqrt{\hat{v}_t^P} \tag{3.9}$$

For the marker model under the Q-distribution we need to estimate the vector of spot variances $\{v_t\}$, and a set of structural parameters $\tilde{\Theta} \equiv \{\kappa, \theta, \lambda, \sigma, \rho\}$. These parameters completely identify the data generating process under the risk-neutral measure. The unobserved spot variance under the Q-measure is filtered from the returns using the PF method as described before, but this time based on the mapped structural parameters $\{\tilde{\kappa}, \tilde{\theta}, \sigma, \rho\}$, where $\tilde{\kappa} = \kappa + \lambda$, and $\tilde{\theta} = \frac{\kappa\theta}{\kappa+\lambda}$. After repeating the same procedure as described before, the Q-measure spot variance on every day can be estimated as the average of the smoothly resampled particles.

$$\hat{v}_t^Q = \frac{1}{N}\sum_{j}^{N} v_t^j \tag{3.10}$$

Similarly, the Q-measure filtered shocks to the index return, conditional on the filtered spot variance would be:

$$\hat{z}_{t+\Delta t}^Q = (ln(S_{t+\Delta t}/S_t) - (\mu - \frac{1}{2}\hat{v}_t^Q))/\sqrt{\hat{v}_t^Q} \tag{3.11}$$

Now define the vega weighted option pricing error of an option n as:

$$\eta_n = (C_n^O - C_n^M(\tilde{\Theta}, \hat{v}^Q))/Vega_n, \quad n = 1, \ldots, M \tag{3.12}$$

where, C_n^O is the observed price of index option n, $C_n^M(\tilde{\Theta}, \hat{v}^Q))$ is the model price for the same option,[11] M is the total number of index options, and $Vega_n$ is the Black-Scholes option vega evaluated at the implied volatility. These vega weighted option pricing errors serve

[11] The time subscript is dropped for compactness.

as an approximation to the implied volatility errors, and since they do not require a numerical inversion of the Black-Scholes model, they are very helpful in large scale optimization problems such as ours. Assuming that these disturbances are i.i.d. normal, the option-based likelihood can be obtained as follows:

$$\ln L^O \propto -\frac{1}{2}\Big(M\ln(2\pi) + \sum_{n=1}^{M}\big(\ln(s^2) + \eta_n^2/s^2\big)\Big) \tag{3.13}$$

where we can replace s^2 by its sample analog $\hat{s}^2 = \frac{1}{M}\sum_{n=1}^{N}\eta_n^2$. The set of structural parameters $\hat{\Theta}$ and $\hat{\tilde{\Theta}}$ can be found as the solution to the following optimization problem:

$$\max_{\Theta,\tilde{\Theta}} \ln L^R + \ln L^O \tag{3.14}$$

3.2.2 Equity Model

We estimate the equity model parameters and the spot idiosyncratic variances for every stock, conditional on the filtered market spot variances and the filtered shocks to the index returns. We want to estimate the set of structural parameters $\Theta_i \equiv \{\mu_i, \kappa_i, \theta_i, \sigma_i, \rho_i, \beta_i\}$ of the model in (2.2), as well as the vector of spot idiosyncratic variances $\{\xi_{i,t}\}$. The Euler discretization of (2.2) yields:

$$\begin{aligned}\frac{(S^i_{t+\Delta t} - S^i_t)}{S^i_t} &= (\mu_i)\Delta t + \beta_i\Big((\mu - r)\Delta t + \sqrt{v_t}z_{t+\Delta t}\Big) + \sqrt{\xi_{i,t}}z_{i,t+\Delta t}\\ \xi_{i,t+\Delta t} &= \xi_{i,t} + \kappa_i(\theta_i - \xi_{i,t})\Delta t + \sigma_i\sqrt{\xi_{i,t}}w_{i,t+\Delta t}\end{aligned} \tag{3.15}$$

As in the case of the market model, for a set of smoothly resampled particles $\{\xi^j_{i,t}\}$ at time t, the P-measure shocks to stock returns $\{z^j_{i,t+\Delta t}\}$ can be obtained conditional on the filtered shocks to market return, $\{\hat{z}^P_{t+\Delta t}\}$.

We then generate correlated shocks to idiosyncratic variance dynamics:

$$w^j_{i,t+\Delta t} = \rho_i z^j_{i,t+\Delta t} + \sqrt{1-\rho_i^2}\epsilon^j_{i,t+\Delta t} \tag{3.16}$$

where, $\epsilon^j_{i,t+\Delta t}$ are independent random variables with mean zero and variance Δt. We can now simulate a raw set of particles, $\{\tilde{\xi}^j_{i,t+\Delta t}\}$ according to equations (3.10), given the set of smoothly resampled particles $\{\xi^j_{i,t}\}$.

$$\tilde{\xi}^j_{i,t+\Delta t} = \xi^j_{i,t} + \kappa_i(\theta_i - \xi^j_{i,t})\Delta t + \sigma_i\sqrt{\xi^j_{i,t}}w^j_{i,t+\Delta t} \tag{3.17}$$

We then have a set of N possible values for $\xi_{i,t+\Delta t}$ to which we want to assign weights. The weight for every particle would be the likelihood that the next day stock return is generated by this particle, given that the next day's index return shock is revealed first.

$$\tilde{W}^j_{i,t+\Delta t} = \frac{1}{\sqrt{2\pi M^j_1}} \cdot exp\Bigg(-\frac{1}{2}\frac{\Big((\frac{S_{i,t+2\Delta t} - S_{i,t+\Delta t}}{S_{i,t+\Delta t}}) - M^j_2 \Big)^2}{M^j_2} \Bigg) \tag{3.18}$$

where M^i_1 and M^j_2 are the conditional mean and variance of the stock return at $t + 2\Delta t$.

$$\begin{aligned} M^j_{1,t+\Delta t} &= E\Bigg[\Big(\frac{S_{i,t+2\Delta t} - S_{i,t+\Delta t}}{S_{i,t+\Delta t}} \Big) \Big| S_{i,t+\Delta t}, \xi^j_{i,t+\Delta t}, \hat{v}_{t+\Delta t}; \hat{z}_{t+2\Delta t} \Bigg] \\ &= \mu_i \Delta t + \beta_i(\mu - r)\Delta t \\ M^j_{2,t+\Delta t} &= var\Bigg[\Big(\frac{S_{i,t+2\Delta t} - S_{i,t+\Delta t}}{S_{i,t+\Delta t}} \Big) \Big| S_{i,t+\Delta t}, \xi^j_{i,t+\Delta t}, \hat{v}_{t+\Delta t}; \hat{z}_{t+2\Delta t} \Bigg] \\ &= (\beta^2_i \hat{v}_{t+\Delta t} + \xi^j_{i,t+\Delta t})\Delta t \end{aligned} \tag{3.19}$$

After normalizing the weights $\tilde{W}^j_{i,t+\Delta t}$ we would have the empirical distribution of the $\xi_{i,t+\Delta t}$, from which we can smoothly resample the next period's particles. We start from $\xi^j_{i,1} = \theta_i \, \forall j$, and repeat this procedure for every day in the sample.

The return-based likelihood functions, which is a function of the P-distribution parameters $\Theta_i \equiv \{\mu_i, \kappa_i, \theta_i, \sigma_i, \rho_i, \beta_i\}$, can be defined as follows:

$$\ln L^R_i \propto \sum_{t=1}^{T} ln\Big(\frac{1}{N} \sum_{j=1}^{N} \tilde{W}^j_{i,t}(\Theta_i) \Big) \tag{3.20}$$

Moreover, the vector of P-measure filtered spot idiosyncratic variances can be obtained as follows:

$$\hat{\xi}^P_{i,t} = \frac{1}{N} \sum_{j}^{N} \xi^j_{i,t} \tag{3.21}$$

For the equity model under the Q-distribution (2.4) we need to estimate the vector of spot variances $\{\xi_{i,t}\}$, and a set of structural parameters $\tilde{\Theta}_i \equiv \{\kappa_i, \theta_i, \lambda_i, \sigma_i, \rho_i, \beta_i\}$. The unobserved spot idiosyncratic variance under the Q-measure is filtered from the returns using the PF method as described before, based on the mapped structural parameters $\{\tilde{\kappa}_i, \tilde{\theta}_i, \sigma_i, \rho_i, \beta_i\}$, where $\tilde{\kappa}_i = \kappa_i + \lambda_i$, and $\tilde{\theta}_i = \frac{\kappa_i \theta_i}{\kappa_i + \lambda_i}$. After repeating the same procedure as described before, the Q-measure spot idiosyncratic variance on every day can be estimated as the average of the smoothly resampled particles.

$$\hat{\xi}_{i,t}^{Q} = \frac{1}{N}\sum_{j}^{N} \xi_{i,t}^{j} \tag{3.22}$$

Given now the market structural parameters $\hat{\tilde{\Theta}}$, estimated market spot variances $\{\hat{v}_t\}$, filtered shocks to the market returns $\{\hat{z}_t^Q\}$, and the estimated Q-measure spot idiosyncratic variances $\hat{\xi}_{i,t}^Q$, we can find the option pricing error for every option as a function of the Q-measure equity structural parameters $\tilde{\Theta}_i$. As with the market model, for an option n written on stock i, we define the vega weighted option pricing error as:

$$\eta_{i,n} = (C_{i,n}^O - C_{i,n}^M(\tilde{\Theta}_i, \hat{\tilde{\Theta}}, \hat{v}^Q, \hat{\xi}_i^Q))/Vega_{i,n}, \quad n = 1, \ldots, M_i \tag{3.23}$$

where, $C_{i,n}^O$ is the observed price of equity option n for stock i, $C_{i,n}^M$ is the model price for the same option, M_i is the total number of options available for stock i, and $Vega_{i,n}$ is the Black-Scholes option vega evaluated at the implied volatility. Assuming that these disturbances are i.i.d. normal, the option-based likelihood can be obtained as follows:

$$\ln L_i^O \propto -\frac{1}{2}\Big(M_i \ln(2\pi) + \sum_{n=1}^{M_i}\big(\ln(s_i^2) + \eta_{i,n}^2/s_i^2\big)\Big) \tag{3.24}$$

where we can replace the s_i^2 by its sample analog $\hat{s}_i^2 = \frac{1}{M_i}\sum_{n=1}^{M_i}\eta_{i,n}^2$. The set of equity structural parameters $\hat{\Theta}_i$ and $\hat{\tilde{\Theta}}_i$ can be found as the solution to the following optimization problem:

$$\max_{\Theta_i, \tilde{\Theta}_i} \ln L_i^R + \ln L_i^O \tag{3.25}$$

3.3 Estimation Results

In this section we report the parameters estimates for the S&P 500 index and the 27 equities in our sample for the period $1996 - 2011$. As explained before, in our estimation we use information from the returns as well as the option prices. For both the index and the equities we use option contracts on each trading day. The index options are European, but the equity options are American. Since the closed form equity option pricing formula in our framework is only available for European options, we eliminate in-the-money options from the sample to avoid biases due to the early premium exercise of American options.[12]

[12] Bakshi et al. (2003) show that the difference between Black-Scholes implied volatilities and American option implied volatilities are negligible for out-of-the-money calls and puts.

3.3.1 Parameter Estimates

Table 4 presents the estimated structural parameters for the market and the equity models. In our joint estimation we restrict the P- and Q-measure parameters to be consistent. So, only the unconditional mean and the speed of mean reversion of the volatility dynamics would be different between the two measures due to the prices of the market and the idiosyncratic volatility risks. Consistent with the previous studies of the market index, we find the price of the volatility risk to be negative, $\lambda = -1.21$. The unconditional mean of the market variance under P and Q are $\theta = 0.037$ and $\tilde{\theta} = 0.061$. Moreover, the speeds of mean reversion of the market volatility dynamics are $\kappa = 3.157$ and $\tilde{\kappa} = 1.94$ under the P and Q dynamics, respectively. These parameter estimates are consistent with those of other studies such as CFJ.

In our estimations we set the market equity risk premium equal to the annual sample average $\mu = 0.078$. Moreover, instead of estimating μ_i for each firm, we run a regression of the equity return on the market excess return, and we set μ_i equal to the OLS alpha of the stock.

The unconditional idiosyncratic variance means are mostly larger than that of the market. Under the physical distribution they range from $\theta_i = 0.034$ for MMM to $\theta_i = 0.146$ for HPQ. The average θ_i for the firms in our sample is 0.068. On the other hand, the speed of mean reversion of equity idiosyncratic variances is much lower than that of the index for all stocks in the sample. It ranges from $\kappa_i = 0.138$ for IBM to $\kappa_i = 1.678$ for AA, with an average of 0.701 for all stocks. The price of idiosyncratic variance risk varies substantially among the firms in our sample, and is of different signs for different stocks. Alcoa has the lowest price of idiosyncratic variance risk $\lambda_i = -0.872$, and XOM has the largest with $\lambda_i = 1.145$. The average absolute value of the price of idiosyncratic variance is 0.213.

Consistent with the literature, $\rho = -0.494$ is large and negative, capturing the so called leverage effect. The correlation between the shocks to equity return and the shocks to idiosyncratic variance is negative for all stocks except for CSCO, HPQ, IBM, MSFT, and UTX. It ranges from $\rho = -0.649$ for JPM to $\rho = 0.1$ for CSCO. Moreover, the beta estimates seems reasonable, ranging from $\beta_i = 0.55$ for MCD to $\beta_i = 1.23$ for AXP. The average beta of the firms in our sample is 0.91.

3.3.2 Filtered Spot Idiosyncratic Variances

As described before, in our estimation we filter the unobserved market variance and the equity idiosyncratic variances from the returns, under both P and Q measures. Table 5 presents the average, standard deviation, minimum, and maximum spot idiosyncratic variance of all firms during the time period in our sample. In the top row we also report the same statistics for the spot index variance.

In Table 6 we present the correlation matrix of the spot idiosyncratic variances, as well as the market spot variance. We can see that there is high degree of correlation between the spot idiosyncratic variances, and all pairwise correlations are positive. The average pairwise correlation between the equity spot idiosyncratic variances is 58%, and the average pairwise correlation of equity spot idiosyncratic variance with market spot variance is 43%. These results show that there is a common structure in idiosyncratic variance levels of equities, and the common factors might be priced. In the following sections, we conduct a systematic analysis of the common structure in equity idiosyncratic variances.

3.4 The Common Structure in Idiosyncratic Volatilities

Several studies have found a common structure in idiosyncratic volatilities[13]. All of these studies, however, have focused on the idiosyncratic volatilities estimated from the returns under the physical distribution. Here we impose consistency in the parameters of the P- and Q-distributions, we verify whether the same common structure exists in the idiosyncratic volatilities estimated form the equity option prices under the Q distribution, and we compare the common factors under both P- and Q-measures.

3.4.1 Implied Idiosyncratic Volatilities

Similar to the implied volatility of an option, we define the implied idiosyncratic volatility ($IIVol$) of an option as the idiosyncratic volatility that would make the model option price equal to the observed price, given the estimated parameters and the estimated spot market volatility. The $IIVol$ can be found as the solution to the following equation.

$$C^{O}_{i,t,n} - C^{M}_{i,t,n}(\hat{\tilde{\Theta}}_i, \hat{\tilde{\Theta}}, \hat{v}^{Q}_{t}, IIVol_{i,t,n}) = 0 \tag{3.26}$$

where, as before, $\hat{\tilde{\Theta}}$ and $\hat{\tilde{\Theta}}_i$ are the estimated Q-measure parameters of the market and equity models, respectively, and $\hat{v}^Q$ is the estimated spot market variance under the risk-neutral measure. For every option written on every stock in our sample we find the $IIVol$ as described above, and we run the following two regressions on every day, one for the implied idiosyncratic volatilities and one for the total implied volatilities.

$$IIVol_{i,n,t} = a^{IVol}_{i,t} + b^{IVol}_{i,t} \cdot (\frac{S_{i,t}}{K_{i,n}}) + c^{IVol}_{i,t} \cdot (DTM_{i,n}) + \epsilon_{i,n,t} \tag{3.27}$$

$$IV_{i,n,t} = a^{IV}_{i,t} + b^{IV}_{i,t} \cdot (\frac{S_{i,t}}{K_{i,n}}) + c^{IV}_{i,t} \cdot (DTM_{i,n}) + \epsilon_{i,n,t} \tag{3.28}$$

Where, i denotes the stock, and n denotes the option contract available for that stock on day t, with strike price $K_{i,n}$ and time to maturity $DTM_{i,n}$. The coefficients, $a^{IVol}_{i,t}, b^{IVol}_{i,t}, c^{IVol}_{i,t}$ in the regression (3.27) represent measures of idiosyncratic volatility level, moneyness slope, and term structure slope, respectively. Moreover, the coefficients, $a^{IV}_{i,t}, b^{IV}_{i,t}, c^{IV}_{i,t}$ in the regression (3.28) represent measures of implied volatility level, moneyness slope, and term structure slope, respectively. So, after estimating the regression coefficients for every stock-day we

[13] See Duarte et al. (2012) and the references within.

have three matrices, $\{a_{i,t}^{IVol}\}$, $\{b_{i,t}^{IVol}\}$, and $\{c_{i,t}^{IVol}\}$, for the implied idiosyncratic volatilities, and three matrices, $\{a_{i,t}^{IV}\}$, $\{b_{i,t}^{IV}\}$, and $\{c_{i,t}^{IV}\}$, for the implied volatilities.

We run a similar regression as in (3.28) for the index options to get the level, slope, and the term structure slope of the market implied volatilities, represented by $\{a_t\}$, $\{b_t\}$, and $\{c_t\}$, respectively.

$$IV_{n,t} = a_t + b_t \cdot (\frac{S_t}{K_n}) + c_t \cdot (DTM_n) + \epsilon_{n,t} \tag{3.29}$$

In what follows, we present a principal component analysis (PCA) of the regression coefficients estimated in (3.27) - (3.29).

Common Structure in Levels

As expected, the PCA of the implied volatility levels indicates a strong factor structure. The first two principal components explain 79% and 11% of the variations across all stocks, respectively, and the first component has a 90% correlation with the market's implied volatility levels. What is surprising is that after accounting for the common market factor in equity returns, there is still a strong factor structure remaining in the implied idiosyncratic volatility levels. Principal component analysis of the $IIVol$ levels shows that the first two principal components explain 58% and 23% of the cross-sectional variations, respectively. Table 7 presents the loadings on the first two components, as well as the percentage of variation captured by each component. The loadings on the first factor are positive for all stocks, while the loadings on the second factor are positive and negative for different stocks. Moreover, both factors are sizeable in terms of explaining the variation, and the fact that the loadings on the second factor take on different signs for different stocks, suggest that it may be related to firm specific characteristics.

The first common component of the $IIVol$ levels has a 99% correlation with the average implied idiosyncratic volatility level of all firms. Further, the first and the second common components have correlations of 65% and 55% with the index implied volatility levels, respectively.

Common Structure in Moneyness Slope

The first two principal components of the implied volatility slopes explain 32% and 8% of the cross-sectional variation, and the first common component has a 38% correlation with the market's implied volatility slope. After removing the common market factor from the returns, there are still commonalities in the moneyness slopes of the implied idiosyncratic volatilities. PCA of the $IIVol$ slopes shows that the first two principal components explain 48% and 6% of the cross-sectional variations, respectively. So the percentage of the variation explained by the first common component of slopes is higher for $IIVol$ than for IV. Table 8 presents the loadings on the first two components, as well as the percentage of variation captured by each component. Similar to those of the levels, the loadings on the first factor are positive for all stocks, while the loadings on the second factor are positive and negative for different stocks, although the signs are not consistent with those of the levels.

The first common component of the $IIVol$ moneyness slopes has a 99% correlation with the average implied idiosyncratic volatility slopes of all firms. Further, the first and the second common components have a correlation of 42% and 8% with the index implied volatility slope, respectively.

Common Structure in Term Structure Slope

The first two principal components of term slope of the implied volatilities explain 57% and 9% of the variations, respectively, and the first principal component has a 78% correlation with the market's term slope. Moreover, the two first principal components of the $IIVol$ term sloes explain 61% and 7% of the variations, respectively. As with the results for the moneyness slope, the proportion of variation explained by the first two principal components of the term slopes are higher for $IIVol$ than IV. Table 9 presents the loadings on the first two components, as well as the percentage of variation captured by each component. Like before, the loadings on the first common component are all positive, while the loadings on the second components have different signs for different stocks.

Moreover, the first principal component has a 99% correlation with the average implied idiosyncratic volatility term slope of all firms. Further, the first and the second common components have a correlation of 78% and -13% with the index implied volatility term slope, respectively.

These results show that after removing the common market factor from the returns, there is still a strong common factor structure in implied idiosyncratic volatilities. This is consistent with the findings of Herskovic et al. (2013) who study the common structure in idiosyncratic volatilities by looking at the firms' fundamentals. Our findings complement the empirical literature that shows there are common factors in the idiosyncratic volatilities of stock returns under the P distribution, by showing that the same factor structure is evident in equity options prices.

3.4.2 Filtered Spot Idiosyncratic Volatilities

So far our results indicate the existence of a common structure in the equity idiosyncratic volatilities obtained from the equity option prices. While there appears to be a market volatility factor in the cross-section of equity idiosyncratic volatilities, the average idiosyncratic volatility of all firms shows stronger correlation with the common components of the implied idiosyncratic volatilities. Moreover, the existence of the common structure is strongest in the levels of the implied idiosyncratic volatilities. Here, we further investigate the common structure in idiosyncratic volatilities by analyzing the spot idiosyncratic volatility levels filtered from the returns.

In our framework, idiosyncratic variance of a stock is a state variable. In section 3.2 we discussed the filtration of this unobserved state variable from the equity returns using the physical and risk-neutral structural parameters, and we presented the properties of these estimated spot idiosyncratic volatilities. Here we perform principal component analysis of the filtered idiosyncratic volatilities under the P and Q distributions. These idiosyncratic variances are estimated from the returns, based on the P- and Q-measure parameters. The spot idiosyncratic volatilities are theoretically the same under the two distributions; however over any discrete interval, such as a day, they would be different due to the price of the idiosyncratic volatility risk.

Principal component analysis of the spot idiosyncratic volatilities under the physical distribution indicates that the first two principal components explain 57% and 30% of the cross-sectional variations, respectively. Table 10 presents the loadings on the first two components for all stocks. The loadings on the first principal component are positive for all stocks, while the loadings on the second principal component have different signs for different stocks. The first principal component has a 98% correlation with the average idiosyncratic volatility. Moreover, the first and the second principal components have correlations of 68% and 48% with the spot market volatilities, respectively.

Principal component analysis of the return-based idiosyncratic volatilities under the Q-distribution yields qualitatively similar results to those under P. The first two principal components explain 55% and 31% of the variations, respectively. The first principal component has a 98% correlation with the average idiosyncratic volatility. Moreover, the first and the second components have a correlation of 67% and -50% with the market spot volatility under the risk-neutral distribution. Table 10 presents the loading on the first two components. The loadings on the first principal component are very close for all stocks under the two distributions. On the one hand, the loadings on the second principal component, while very close in absolute value, are of the exact opposite sign under the P- and Q-distributions. In other words, the second principal component under the P measure is almost perfectly and negatively correlated with that under the Q measure. This estimation is very similar to the one done in 3.4.1, and it is not surprising that the results are very similar. Indeed, the data used for the PCA here is the same one that was used to generate the model parameters, which in turn were used to generate the data for the PCA of 3.4.1. The interest of Section 3.4.1 is that it shows that the commonality exists across slope and term structure.

The average idiosyncratic volatility seems to explain the cross-sectional variation of the idiosyncratic volatilities very well, and it is highly correlated with the market volatility, which is also highly correlated with the first two components of the idiosyncratic volatilities. We regress the market spot variance on the average idiosyncratic variances to find the component of the market variance that is orthogonal to the average idiosyncratic variance. The R^2 of the regression is 30% and 26%, under the P and Q distributions, respectively. Moreover, the vector of the residuals, which is the component of the market variance that is orthogonal to the average idiosyncratic variance, denoted by F_P^{orth}, has a 69% correlation with the second principal component of the idiosyncratic variances. Under the Q distribution, the orthogonal component, denoted by F_Q^{orth}, has a -71% correlation with the second principal component. Our results suggest that the average idiosyncratic variance and the component of the market spot variance that is orthogonal to it are good proxies for the common principal components of the equity idiosyncratic volatility levels.

4 Cross Section of Equity Returns

The previous sections show that there is a common factor structure in the idiosyncratic volatilities. The existence of common factors among equity idiosyncratic volatilities does not necessarily mean that idiosyncratic volatility is priced. If however, there are one or more systematic risk factors missing from the model, the exposure to those factors would be captured by the idiosyncratic volatilities, and the common components of idiosyncratic volatilities are related to the variances of the missing systematic factors. In our one factor model, the equity return is represented as follows:

$$r_i = \mu_i + \beta_i(r_m - r_f) + \epsilon_i \tag{4.1}$$

If the market excess return is not the only systematic risk factor and there are K factors, $F_1, \ldots, F_K$, missing from the model, then the residuals in (4.1) are in fact equal to:

$$\epsilon_i = \beta_{i,1}F_1 + \ldots + \beta_{i,K}F_K + u_i \tag{4.2}$$

where, u_i is the true idiosyncratic residual. So, the idiosyncratic variance, as defined in a one factor model, is related to the variance of the missing factors and their corresponding loadings, as follows:

$$var(\epsilon_i) = \beta_{i,1}^2 \cdot var(F_1) + \ldots + \beta_{i,K}^2 \cdot var(F_K) + var(u_i) \tag{4.3}$$

Moreover, if idiosyncratic volatility is priced due to exposure to the missing factors, we would expect the common components of the equity idiosyncratic volatilities to help explain the cross-section of equity returns. This is what we investigate in this section. In particular, we test whether the average idiosyncratic variance, AIV, that we found to proxy for the first principal component of the idiosyncratic variances has any explanatory power in explaining the cross section of the 25 Fama-French portfolios, which are formed on size and book-to-market. We follow the standard two-step procedure for cross-sectional asset pricing. First, we run a time series regression of the excess portfolio returns on the Fama-French factors, as well as the proposed AIV factor for each portfolio.

$$r_{p,t}^e = a_p + b_p^m \cdot (r_t^m - r_t) + b_p^{smb} \cdot r_t^{SMB} + b_p^{hml} \cdot r_t^{HML} + b_p^{AIV} \cdot AIV_t + \varepsilon_{p,t} \tag{4.4}$$

where, $r_{p,t}^e$ is the excess return of portfolio p at time t, $(r^m - r)$ is the excess market return, and r^{SMB} and r^{HML} are the returns on the size (small-minus-big) and value (high-minus-low) factors, respectively.[14] Moreover, AIV is the average idiosyncratic variance of the firms in our sample, and is estimated as described in the previous sections. Figure 1 presents the coefficients of the AIV factor in the time series regressions (4.4). The coefficients of AIV are

[14]Data on these portfolios and factors are downloaded from Kenneth French's website.

all positive, and even in the presence of the Fama-Franch factors, are significantly different from zero for 20 out of the 25 portfolios.

In the second step, we regress the average portfolio excess returns on the time series coefficient estimates:

$$\bar{r}_p^e = \gamma_0 + \gamma_m \cdot \hat{b}_p^m + \gamma_{smb} \cdot \hat{b}_p^{smb} + \gamma_{hml} \cdot \hat{b}_p^{hml} + \gamma_{AIV} \cdot \hat{b}_p^{AIV} + \varepsilon_p \tag{4.5}$$

In order to compare the explanatory power of AIV to that of the Fama-French factors, we use different linear combinations of the factors in (4.5). In Table 11 we present the regression results. As we see in the table, the AIV factor premium is significantly and economically different from zero. Moreover, when added to the Fama-French model, the AIV factor reduces the pricing error from 0.137% per day to 0.11%, and it increases the adjusted R^2 from 82% to 84%. Further, the combination of the market factor and the AIV factor performs better than the combination of market and SMB, as well as the market and HML, in terms of explaining the cross sectional variations of the 25 portfolios' returns. It is worth noting that our sample contains only 27 equities, and our definition of the average idiosyncratic variance is rather narrow. We would expect that as the sample size gets larger, the AIV factor would be able to better explain the cross section of equity returns.

In our sample the risk premium of SMB is not significantly different from zero in any of the regressions. However, HML has a positive and significant risk premium. The AIV also carries a positive risk premium equal to 0.01% per day. Moreover, the fact that AIV does not drive out the HML in the regression where all factors are present, combined with the fact that SMB has an insignificant risk premium in all regressions, suggests that the explanatory power of AIV comes from exposure to missing systematic risk factors other than SMB and HML.

5 Idiosyncratic Variance Risk Premium

So far we have shown that equity idiosyncratic variance is priced, and that there is factor structure among idiosyncratic variances. Moreover, we showed that the common component of idiosyncratic variances is a priced factor in the cross section of equity returns, and that it can reduce the pricing error when added to the Fama-French factors. In this section we discuss how we can create portfolios that are only exposed to the idiosyncratic variance risk of an equity, and we present evidence that for the majority of the equities in our sample, the mean return of these portfolios is significantly different from zero. Moreover, we present a measure of idiosyncratic variance risk premium, and we investigate whether this risk premium is explained by the usual equity risk factors.

5.1 Evidence from Portfolio Returns

Equity options are investment assets, and their expected returns are of particular interest to practitioners and academics. In a stochastic volatility framework, the underlying price and its volatility are state variables, and the expected index option return depends on the risk premiums associated with them. In our framework, idiosyncratic volatility is also a state variable, and since we show that it is priced, its risk premium would be a part of the expected equity option return. The following proposition provides an expression for the expected index and equity option returns under the physical distribution.

Proposition 2. *For a derivative $f(t, S, v)$ written on the index with price S and variance v at time t, the instantaneous expected excess return on the derivative contract is given by:*

$$\frac{1}{dt}E_t^P[\frac{df}{f} - rdt] = f_s\frac{S_t}{f}(\mu - r) + f_v\frac{1}{f}\lambda v_t \tag{5.1}$$

and for a derivative $f^i(t, S_i, v_i)$ written on the equity with price S_i and total variance v_i at time t, the instantaneous expected excess return on the derivative contract is given by:

$$\frac{1}{dt}E_t^P[\frac{df^i}{f^i} - rdt] = f^i_{s_i}\frac{S_{i,t}}{f^i}\big((\mu_i - r) + \beta_i(\mu - r)\big) + f^i_{v_i}\frac{1}{f^i}(\beta_i^2\lambda v_t + \lambda_i\xi_{i,t}) \tag{5.2}$$

where, f_s, f_v, $f^i_{s_i}$, and $f^i_{v_i}$ represent partial derivatives, and the structural parameters and state variables are as defined before.

Proof. See Appendix B. □

So, the expected excess return of an equity option depends on the equity and variance risk premiums of the index through the equity beta, as well as on the idiosyncratic return and idiosyncratic variance risk premiums.

Now consider a delta hedged portfolio of an index option and the index, denoted by π. Similarly we construct a delta hedged portfolio of an equity option and the underlying stock, denoted by π^i. These portfolios are by construction insensitive with respect to the changes in the underlying asset's price. Using the results of Proposition 2, the instantaneous expected excess return on these delta neutral portfolios can be shown to be the following.

$$\frac{1}{dt}E_t^P[\frac{d\pi_t}{\pi_t} - rdt] = f_v\frac{1}{\pi_t}\lambda v_t \tag{5.3}$$

$$\frac{1}{dt}E_t^P[\frac{d\pi^i_t}{\pi^i_t} - rdt] = f^i_{v_i}\frac{1}{\pi^i_t}(\beta_{i,t}^2\lambda v_t + \lambda_i\xi_{i,t}) \tag{5.4}$$

The portfolio π's return depends only on the market variance risk premium, while the return of portfolio π^i depends on the market variance risk premium through the equity beta, as well as on the idiosyncratic variance risk premium. So we can create a portfolio by taking positions in π and π^i, that is insensitive with respect to the market variance, and is only exposed to the idiosyncratic variance risk. Consider a hedge portfolio, Π, that consists of y units of a delta hedged index option π, and x units of a delta hedged equity options π^i. The portfolio value at any time is the following:

$$\Pi_t = x_t \cdot \pi^i + y_t \cdot \pi_t \tag{5.5}$$

The instantaneous expected excess return of this portfolio can be found directly from (5.3) and (5.4), and it is equal to:

$$\frac{1}{dt} E_t^P [\frac{d\Pi_t}{\Pi_t} - rdt] = \Big((x_t f_{v_i}^i \beta_{i,t}^2 + y_t f_v) \lambda v_t + x_t f_{v_i}^i \lambda_i \xi_{i,t} \Big) \frac{1}{\Pi_t} \tag{5.6}$$

Our goal is to choose x and y at any time so that the portfolio Π is not exposed to the market variance risk premium. This can be accomplished if x and y at any time have the following relationship.

$$y_t = -x_t \frac{f_{v_i}^i}{f_v} \beta_{i,t}^2 \tag{5.7}$$

At any time t we choose $x_t = \frac{1}{f_{v_i}^i}$ and $y_t = \frac{-1}{f_v} \beta_{i,t}^2$. So, according to (5.6) the instantaneous expected excess return on our hedge portfolio would be:

$$\frac{1}{dt} E_t^P [\frac{d\Pi_t}{\Pi_t} - rdt] = \lambda_i \xi_{i,t} \frac{1}{\Pi_t} \tag{5.8}$$

Therefore, a portfolio with a long position in $\frac{1}{f_{v_i}^i}$ units of a delta neutral equity option, and a short position in $\frac{\beta_{i,t}^2}{f_v}$ units of a delta neutral index option is instantaneously insensitive with respect to the changes in the equity price, index price, and the market variance. The only risk that this portfolio bears is the idiosyncratic variance risk of the equity, and the return on this portfolio would be the idiosyncratic volatility risk premium.

In what follows, we describe how we form and rebalance portfolios that only loads on the idiosyncratic variance risk premiums of equities, and we investigate whether the return on these portfolios are statistically and economically different from zero. For every firm in our sample, as well as for the market index, we create hedge portfolios using options with 30 days to maturity, and with three moneyness rations 1, 1.025, and 1.05 (1, 0.975, and 0.95) for calls (puts), respectively. The detail description of the data used and the way the portfolios are formed and rebalanced are presented in Appendix C. If the equity idiosyncratic variance bears a risk premium, we would expect these portfolios to have mean returns that are significantly different from zero.

5.1.1 Portfolio Returns

In tables 12 and 13 we present the mean annualized returns of the hedge portfolios for calls and puts, respectively. Present in the tables are also the t-statistics for the null hypothesis that mean portfolio return is zero. These portfolios are by construction only exposed to the idiosyncratic variance risk of the equity, and a significant mean portfolio return is an indication of a non-zero idiosyncratic volatility risk premium. There is quite a bit of variation across call and put portfolios, and across different moneyness rations. For call portfolios, the mean returns are of different signs for different equities, and they generally decrease as the moneyness increases. In the last row of the table we also report the number of firms for which the mean portfolio return is significantly different zero. For call portfolios, as we move further away from the money, the number of firms with significant idiosyncratic volatility risk premium decreases.

Put portfolios also indicate positive and negative idiosyncratic volatility risk premiums for different stocks. The number of stocks with significant idiosyncratic volatility risk premium, as evident from the mean put portfolio returns, is large for any moneyness ratio. In general, the results obtained from the put portfolios are more indicative of the significant equity idiosyncratic risk premium.

It should be noted that, in our portfolio rebalancing we do not take into account the transaction costs. We merely provide these results as indication of non-zero idiosyncratic volatility risk premium, and our portfolio formation and rebalancing is a statistical procedure for highlighting this evidence, rather than an implementable trading strategy. Motivated by these results, in the next section, we proceed with introducing a measure of idiosyncratic volatility risk premium, and we analyze the properties of these risk premiums for the firms in our sample.

5.2 Measure of Idiosyncratic Volatility Risk Premium

Here we present a measure of idiosyncratic variance risk premium, and investigate whether is it statistically and economically significant, and whether it can be explained by the usual equity risk factors. The instantaneous idiosyncratic variance risk premium in our modeling framework is:

$$E_t^P[d\xi_t] - E_t^Q[d\xi_t] = \lambda_i \xi_t \tag{5.9}$$

The idiosyncratic variance risk premium (RP) at any time t over a discrete time interval of length $T - t$ can be obtained as the difference between the expected integrated idiosyncratic variance under physical and risk-neutral distributions.

$$IVRP_{i,t} = \frac{1}{T-t}E_t^P\Big[\int_t^T \xi_{i,s}ds\Big] - \frac{1}{T-t}E_t^Q\Big[\int_t^T \xi_{i,s}ds\Big]$$
$$= \Big(\theta_i + \frac{1-e^{-\kappa_i(T-t)}}{\kappa_i(T-t)}(\xi_{i,t}^P - \theta_i)\Big) - \Big(\tilde{\theta}_i + \frac{1-e^{-\tilde{\kappa}_i(T-t)}}{\tilde{\kappa}_i(T-t)}(\xi_{i,t}^Q - \tilde{\theta}_i)\Big) \quad (5.10)$$

Where, $IVRP_{i,t}$ is the idiosyncratic variance RP of stock i, ξ_t^P and ξ_t^Q are the estimated spot idiosyncratic variances at time t under P and Q, and the rest of the structural parameters are as defined before. In our calculations we choose T to be 30 days, and we calculate the annualized 30-day idiosyncratic variance RP. We also calculate the 30-day market variance risk premium as:

$$MVRP_t = \frac{1}{T-t}E_t^P\Big[\int_t^T v_s ds\Big] - \frac{1}{T-t}E_t^Q\Big[\int_t^T v_s ds\Big]$$
$$= \Big(\theta + \frac{1-e^{-\kappa(T-t)}}{\kappa(T-t)}(v_t^P - \theta)\Big) - \Big(\tilde{\theta} + \frac{1-e^{-\tilde{\kappa}(T-t)}}{\tilde{\kappa}(T-t)}(v_t^Q - \tilde{\theta})\Big) \quad (5.11)$$

Where, $MVRP_{i,t}$ is the market variance RP and, v_t^P and v_t^Q are the estimated spot market variances at time t under P and Q. The descriptive statistics for the idiosyncratic variance RP of each stock, as well as the market volatility RP, are presented in Table 14. We also report in the last column the t-stat of the null hypothesis that the average idiosyncratic variance RP is zero. Consistent with the estimation results for the structural parameters, the average idiosyncratic variance of the stocks in our sample can take positive or negative signs. Moreover, the mean RP is statistically different from zero for all stocks. Earlier studies have found that the market variance RP is negative. Consistent with these previous results[15], the average market variance RP in our sample is -0.48% per annum, and it is strongly significantly different from zero.

Principal component analysis of the idiosyncratic variance RP's shows that the first two principal components explain 70% and 20% of the variation, respectively. These results were expected, since the RP's reflect the difference between the P- and Q-estimates of the levels of idiosyncratic variances, for both of which there is a high commonality as we saw before. The first principal component has a 89% correlation with the average idiosyncratic variance risk premium defined as follows.

$$AIVRP_t = \frac{1}{N}\sum_{i=1}^{N} IVRP_{i,t} \quad (5.12)$$

The first principal component of the idiosyncratic variance risk premiums is also highly correlated with the market variance risk premium with the correlation coefficient of -0.78%. We regress the market variance risk premium on to the average idiosyncratic variance risk premium. With $R^2 = 75\%$, the average idiosyncratic variance risk premium is a strong predictor

[15] See Carr and Wu (2009), and Driessen et al. (2009).

of the market variance risk premium. Moreover, the residuals of the regression, denoted by F_{RP}^{orth} , which is the component of market variance risk premium that is orthogonal to the $AIVRP$ has a correlation of 45% with the second principal component of the idiosyncratic variance risk premiums. These results are qualitatively and quantitatively consistent with those presented in the previous section.

5.2.1 Are Idiosyncratic Variance Risk Premiums Explained by the Usual Equity Risk Factors?

In our modeling framework, the independent variation in idiosyncratic variance represents an additional source of risk, independent from the equity return risk premium that is due to covariation of the equity return and market return. Under the classical CAPM and its extensions, idiosyncratic variance risk premium cannot come from an independent source of risk, and can only come from the correlation between the idiosyncratic variance and the market return. In this section we investigate whether the Fama-French three factors plus the momentum factor can explain the variation in the idiosyncratic variance risk premiums. For every firm in our sample we run the following regression:

$$IVRP_{i,t} = a_i + b_i^m \cdot (r_t^m - r_t) + b_i^{smb} \cdot r_t^{SMB} + b_i^{hml} \cdot r_t^{HML} + b_i^{mom} \cdot r_t^{mom} + \varepsilon_{i,t} \tag{5.13}$$

where, $IVRP_{i,t}$ is the idiosyncratic variance risk premium of stock i at time t, and the rest of the factors are as defined before. Table 15 presents the OLS coefficient estimates, the t-stats, and the R^2 of the regressions. As is evident from the R^2, these equity risk factors cannot explain the time-series variations in the idiosyncratic variance risk premiums. Moreover, the coefficients of the market return and SMB factors are not significantly different from zero for almost all of the stocks.

Idiosyncratic variance risk premium can also come from the correlation between the idiosyncratic variance and the market variance. In a stochastic volatility model such as Heston (1993), a negative market variance risk premium is generated because of the negative correlation between the market variance and market return. Christoffersen et al. (2013) present a pricing kernel in which both the equity premium and the variance premium have two distinct components originating in preferences. So, if the idiosyncratic variance of a firm's equity is correlated with aggregate volatility, then the market variance risk premium should be able to explain at least part of the variation in idiosyncratic variance risk premiums. To test this conjecture we run the following regression:

$$IVRP_{i,t} = a_i + b_i \cdot MVRP_t + \varepsilon_{i,t} \tag{5.14}$$

The results are presented in Table 16. The coefficient of the $MVRP$ is significant for all stocks except for JNJ. The R^2's are much larger compared to the previous regression, ranging from zero for JNJ to 82% for CVX, and with the average R^2 equal to 26%. Moreover, the coefficient of the $MVRP$ is positive (negative) for stocks with negative (positive) average idiosyncratic variance risk premium.

Motivated by our PCA results in the previous sections, we also test the explanatory power of the average idiosyncratic variance risk premium, $AIVRP$, together with the component of the market variance risk premium orthogonal to it (F_{RP}^{orth}) in the following regression:

$$IVRP_{i,t} = a_i + b_{AIVRP,i} \cdot AIVRP_t + b_{orth,i} \cdot F_{RP}^{orth} + \varepsilon_{i,t} \tag{5.15}$$

The two factors can significantly improve the R^2 compared to the previous regressions. The R^2 now ranges from 10% for KO to 89% for CVX, with the average R^2 equal to 51%. Moreover, the coefficients as well as the intercept are significantly different from zero for all stocks. The results are presented in Table 17.

6 Conditional Equity Betas

Stock betas are one of the most important measures of equity risk. The importance of stock betas in corporate finance and asset pricing has motivated researchers to look for better methods of estimating these variables. An accurate measurement of betas is crucial in many applications such as cost of capital estimation and detection of abnormal returns. Stock betas are usually estimated using historical returns on the stock and the market index. There is a consensus that stock betas are time varying, and the popular approach to account for the time variation is to use a rolling window of historical returns. There are other sophisticated estimation methods based on historical betas to capture the time variation. All these techniques are based on historical information, and the main premise is that the future will be similar to the past.

Other proposed methods of estimating the equity betas use the information inherent in option prices. Option prices contain information about the future distribution of the underlying asset, and incorporating this information can potentially lead to better estimates for any variable, especially when the historical patterns are unstable or when there have been structural breaks.

In estimating stock betas from option prices, a few important issues should be noted. First, the information inherent in option prices is related to the risk neutral distribution of the underlying asset. Since betas are ultimately used as a measure of equity risk under the physical measure, a proper link should be made between the P and Q distributions, and the premiums for the priced risks in the market should be taken into account.

Second, the consistency between the index option prices and equity option prices as well as the consistency between the market's P and Q distributions are very important. Driessen et al. (2009) study the relationship between equity options and index options, and they find structural differences which they explain by the existence of correlation risk. Bates (2000) indicates that any successful option pricing model should be able to reconcile the P and Q distributions of the underlying asset. Constantinides et al. (2011) show widespread evidence of mispricing in the index options resulting from the lack of consistency between the index option and the underlying markets.

Several studies have proposed to use the option implied information in estimating betas. French, French et al. (1983) compute betas based on historical correlations and the implied volatilities of the market and the equity. Siegel (1995) proposes the creation a derivative from which implied betas can be estimated. Perhaps most related to our study are two recent papers by Chang et al. (2011), hereafter CCVJ, and Buss and Vilkov (2012), hereafter BV.

BV use implied volatilities of the market and equity along with a parametric model for implied correlations, and estimate the betas. CCVJ show that in a one factor model, and under the assumption of zero skewness for the idiosyncratic returns, the implied betas can be estimated as the product of the ratio of equity to market implied skewness, and the ratio of equity to market implied volatilities. They derive option implied moments of the equity risk neutral distribution based on the method of Bakshi et al. (2003) using only one day of observed option prices, and their estimation does not rely on historical returns.

All these papers estimate the equity beta as the product of the correlation between the market and equity returns and the ratio of equity to market volatility, and the main distinguishing feature among them is how the correlation is estimated. In this paper the equity betas appear in equation (2.2) in the model and are estimated as part of the structural parameters in our framework. The advantages of our beta estimates are as follows. First, since the beta enters both the P- and Q-equity return dynamics explicitly, it is estimated directly, without making any assumptions regarding the correlations. Moreover, since our estimation methodology is based on the joint information in the stock returns and equity option prices, our beta estimates take all available information from the P and Q distributions into account. In Table 18 we present the estimated betas for the firms in our sample, based on the information in returns alone, based on the information in option prices alone, and based on the joint information. We also report the OLS betas.

Second, since we use an equity option pricing model that links the equity price dynamics to the market price dynamics, the consistency between the index option market and equity option market is taken into account. Third, our option pricing model is based on the assumptions regarding the physical dynamics of the equity price, and the explicit transformation to the risk neutral measure. So, betas are estimated taking into account the price of market variance risk, as well as the price of idiosyncratic variance risk. The methods in the previous studies that estimate the betas from the option markets use exclusively the RN measure; since we are ultimately interested in the application of these betas under the physical measure, these estimates might be biased because of the presence of the market risk premiums.

In the model presented in this paper the betas are assumed to be constant. There is, however, widespread agreement in the literature that equity betas are time varying. The reason that betas change over time is the time variation in market volatility, stock volatility, and the correlation between the market and stock returns. To capture the time variation in equity betas and account for the conditionality, we propose the following procedure to estimate betas.

We fix the risk neutral structural parameters, $\hat{\hat{\Theta}}$ and $\hat{\hat{\Theta}}_i$, that we estimated before for the market dynamics and equity dynamics. Our parameter estimates are based on the full sample of option prices for the index and the equities. Since the data generating process for the stochastic volatility process of the market return and idiosyncratic return are assumed to remain the same over time, and since the parameters are constant, we use the entire sample to estimate these parameters to increase estimation precision.

On every day t in our sample and given the structural parameters of the market and of every equity, excluding the constant beta estimate, and given the estimated spot market variance, we can find the conditional beta and the spot idiosyncratic variance for every equity i using the equity option prices observed at time t:

$$\{\hat{\xi}_{i,t}, \hat{\beta}_{i,t}\} = \underset{\{\xi_{i,t},\beta_{i,t}\}}{\operatorname{argmax}} \sum_{n=1}^{M_{i,t}} (C^O_{i,t,n} - C^M_{i,t,n}(\hat{\hat{\Theta}}_i, \hat{\hat{\Theta}}, \hat{v}^Q_t, \xi_{i,t}))^2 / Vega^2_{i,t,n}, t = 1, 2, \ldots, T,\ i = 1, \ldots, N. \quad (6.1)$$

Where, as before, $C^O_{i,t,n}$ is the observed price, $C^M_{i,t,n}$ is the model price, and M_i is the number of option contract with six months to maturity available for stock i at time t. The options used in estimating the conditional betas are not in the sample that we used for our estimation in previous sections. So, our estimation of betas is done out-of-sample. The choice of a six-month horizon is to create a balance between the option liquidity that is largest for short maturity, and the relevant horizon for firm risk, which is arguably considerably longer. Moreover, given the estimated market and equity structural parameters, and given the spot market variance, the estimation of beta on any day relies on the observed options on that day alone. This feature is similar to that in CCJV, and allows for more reliable beta estimates when new information is released about the firm.

In Table 19 we repost the mean, standard deviation, minimum and maximum of the conditional betas for each stock. In the last column we also present the unconditional betas estimated before for comparison. The mean conditional betas are larger than the unconditional ones for most of the firms. The average is 1.1 across the 27 stocks compared to the average of 0.91 for the unconditional betas. Nonetheless, in almost all cases the mean of the conditional betas lies within one standard deviation of its unconditional value.

7 Conclusion

We use a one factor model for equity return dynamics in which the idiosyncratic volatility of the stock follows a stochastic process, and is allowed to be priced. We develop a method to estimate the structural parameters as well as the spot idiosyncratic variances using the return data and the option data. Our estimation is based on a joint likelihood function that has a return component and an option component, while the structural parameters are internally consistent between the physical and risk neutral measures. In a recent study Christoffersen et al (2013) use the same model, but they assume that the idiosyncratic variance is not priced. This assumption implies that the market return is the only priced risk factor, and that the one factor model is the correct model. In our estimation we show that the price of idiosyncratic variance is significantly different from zero for all the 27 firms in our sample.

We calculate the implied idiosyncratic variance of the options in our sample, and we show that even after removing the market factor from equity returns, there is still a strong factor structure in implied idiosyncratic variances. Our principal component analysis shows that there is a strong factor structure in option implied levels, moneyness slopes, and term structure slopes of idiosyncratic variances. We show that the average idiosyncratic variance, AIV, of all stocks is a good proxy for the common factor. These findings complement the literature that documents the existence of a common factors structure among idiosyncratic variances under the physical distribution.

We show that if there are factors missing from our one factor model, the idiosyncratic variance as defined in a one factor model, captures the exposure to the variance of the missing factors. We show that the AIV factor has positive loadings in the time series regression of the 25 Fama-French value and size portfolios. Moreover, we show that the AIV factor reduces the cross sectional pricing error of these portfolios when added to the Fama-French model. The AIV has a significant and positive risk premium, and its cross sectional explanatory power, in our sample period, is distinct from and more than that of the HML and SMB factors.

We derive the expected option return in our framework, and we discuss a trading strategy involving the equity option, the underlying equity, the index option, and the index. These portfolios are constructed and rebalanced in such a way that they are only exposed to the idiosyncratic variance risk of the equity. We show that the mean returns on these portfolios are significantly different from zero for the majority of the equities in our sample, which is an indication of the existence of a premium for the idiosyncratic variance risk.

We propose a measure for the idiosyncratic variance risk premium, defined as the difference between the P and Q expected integrated idiosyncratic variance. We show that the mean idiosyncratic variance risk premium is significantly different from zero for the firms in our sample, and that this risk premium is not explained by the usual risk factors. Moreover, we show that the time series variations of idiosyncratic variance risk premiums are well explained by the average idiosyncratic variance risk premium together with the component of the market variance risk premium orthogonal to the average idiosyncratic variance risk premium.

Finally, we discuss the implications of our model for the estimation of equity betas. The equity beta in our model is estimated as part of the structural parameters, using the simultaneous information from returns and options. Moreover, we propose a method to characterize the time variation in equity betas.

Appendix

A Proof of Proposition 1

The market index is assumed to follow the stochastic volatility model below:

$$\begin{aligned} dS_t/S_t &= (\mu)dt + \sqrt{v_t}dz_t, \\ dv_t &= \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}dw_t \end{aligned} \tag{A.1}$$

Using Girsanov's theorem we can write the following transformation for the two Brownian motions dz and dw.

$$\begin{aligned} dz &= d\tilde{z} - (\psi_1 + \rho\psi_2)dt, \\ dw &= d\tilde{w} - (\rho\psi_1 + \psi_2)dt \end{aligned} \tag{A.2}$$

where, ψ_1 and ψ_2 are the price of risk for dz and dw, respectively, and ρ is the correlations between the two Brownian motions. The drift of the index return dynamics under the risk neutral measure is equal to the risk free rate, so we have the following restriction:

$$\psi_1 + \rho\psi_2 = \frac{\mu - r}{\sqrt{v}} \tag{A.3}$$

Moreover, we assume that the price of volatility risk is proportional to volatility. So we have the second restriction as follows:

$$\rho\psi_1 + \psi_2 = \frac{\lambda\sqrt{v}}{\sigma} \tag{A.4}$$

The unique prices of risk can be found from (A.3) and (A.4), and are the following:

$$\begin{aligned} \psi_1 &= \frac{\mu\sigma - \rho\lambda v}{\sigma\sqrt{v}(1-\rho^2)}, \\ \psi_2 &= \frac{\lambda v - \mu\rho\sigma}{\sigma\sqrt{v}(1-\rho^2)} \end{aligned} \tag{A.5}$$

Replacing (A.2)-(A.5) into (A.1) yields the index return dynamics under the risk neutral measure.

$$\begin{aligned} dS_t/S_t &= rdt + \sqrt{v_t}d\tilde{z}_t, \\ dv_t &= \tilde{\kappa}(\tilde{\theta} - v_t)dt + \sigma\sqrt{v_t}d\tilde{w}_t \end{aligned} \tag{A.6}$$

where, $\tilde{\kappa} = \kappa + \lambda$, and $\tilde{\theta} = \frac{\kappa\theta}{\kappa+\lambda}$.

For the equity return dynamics we have the following dynamics under the physical measure:

$$\begin{aligned} dS_{i,t}/S_{i,t} &= (\mu_i)dt + \beta_i(dS_t/S_t - rdt) + \sqrt{\xi_{i,t}}dz_{i,t}, \\ d\xi_{i,t} &= \kappa_i(\theta_i - \xi_{i,t})dt + \sigma_i\sqrt{\xi_{i,t}}dw_{i,t} \end{aligned} \tag{A.7}$$

Representing the prices of idiosyncratic shocks dz_i and dw_i by ψ_1^i and ψ_2^i, respectively, we have the following transformation using Girsanov's theorem:

$$\begin{aligned} dz_i &= d\tilde{z}_i - (\psi_1^i + \rho_i\psi_2^i)dt, \\ dw_i &= d\tilde{w}_i - (\rho_i\psi_1^i + \psi_2^i)dt \end{aligned} \tag{A.8}$$

Similar to the case of the market return dynamics, we apply the following restrictions to the prices of risk, assuming that the price of idiosyncratic volatility risk is proportional to idiosyncratic volatility.

$$\begin{aligned} \psi_1^i + \rho_i\psi_2^i &= \frac{\mu_i - r}{\sqrt{\xi_i}}, \\ \rho_i\psi_1^i + \psi_2^i &= \frac{\lambda_i\sqrt{\xi_i}}{\sigma_i} \end{aligned} \tag{A.9}$$

solving for ψ_1^i and ψ_1^i we have the following prices of idiosyncratic risk.

$$\begin{aligned} \psi_1^i &= \frac{(\mu_i - r)\sigma_i - \rho_i\lambda_i\xi_i}{\sigma_i\sqrt{\xi_i}(1-\rho_i^2)}, \\ \psi_2^i &= \frac{\lambda_i\xi_i - (\mu_i - r)\rho_i\sigma_i}{\sigma_i\sqrt{\xi_i}(1-\rho_i^2)} \end{aligned} \tag{A.10}$$

Replacing (A.8)-(A.10) into (A.7) we have the following equity return dynamics under the risk neutral measure:

$$\begin{aligned} dS_{i,t}/S_{i,t} &= rdt + \beta_i(dS_t/S_t - rdt) + \sqrt{\xi_{i,t}}d\tilde{z}_{i,t}, \\ d\xi_{i,t} &= \tilde{\kappa}_i(\tilde{\theta}_i - \xi_{i,t})dt + \sigma_i\sqrt{\xi_{i,t}}d\tilde{w}_{i,t} \end{aligned} \tag{A.11}$$

where, $\tilde{\kappa}_i = \kappa_i + \lambda_i$, and $\tilde{\theta}_i = \frac{\kappa_i\theta_i}{\kappa_i+\lambda_i}$. □

B Proof of Proposition 2

We derive the instantaneous return for the equity option. The return for index option can be derived similarly. Let $f^i(t, S_i, v_i)$ be the price of a derivative whose price depends on the spot price and spot variance of the equity. Applying Ito's lemma to $f^i(t, S_i, v_i)$, we have:

$$df^i = f^i_t dt + f^i_{s_i} dS_i + \frac{1}{2} f^i_{s_i s_i} dS_i dS_i + f^i_{v_i} dv_i + \frac{1}{2} f^i_{v_i v_i} dv_i dv_i + f^i_{s_i v_i} dS_i dv_i \tag{B.1}$$

where, f^i_x and f^i_{xy} denote the first and second partial derivative of f with respect to x and xy, respectively. Moreover, in our one factor model, total variance of the equity return is defined as $v_i = \beta_i^2 v + \xi_i$. Using (2.1)-(2.4) together with the pricing PDE, we have the following equation for df^i under the physical measure:

$$df^i = \Big(rf^i - rS_i f^i_{s_i} - f^i_{v_i} \Big(\beta_i^2 \tilde{\kappa}(\theta - v) + \tilde{\kappa}_i(\tilde{\theta}_i - \xi_i) \Big) \Big) dt + f^i_{s_i} dS_i + f^i_{v_i} dv_i \tag{B.2}$$

Note that our model implies that:

$$\begin{aligned} \frac{1}{dt} E^P_t[dS_i] &= \Big(\mu_i + \beta_i(\mu - r) \Big) S_i, \\ \frac{1}{dt} E^P_t[dv_i] &= \beta_i^2 \kappa(\theta - v) + \kappa_i(\theta_i - \xi_i) \end{aligned} \tag{B.3}$$

Relations (B.3) together with (B.2) yields the following:

$$\frac{1}{dt} E^P_t[\frac{df^i}{f^i} - rdt] = \frac{f^i_{s_i}}{f^i} (\frac{1}{dt} E^P_t[dS_i] - rS_i) + \frac{f^i_{v_i}}{f^i} \Big(\frac{1}{dt} E^P_t[dv_i] - \Big(\beta_i^2 \tilde{\kappa}(\tilde{\theta} - v) + \tilde{\kappa}_i(\tilde{\theta}_i - \xi_i) \Big) \Big) \tag{B.4}$$

which simplifies to:

$$\frac{1}{dt} E^P_t[\frac{df^i}{f^i} - rdt] = f^i_{s_i} \frac{S_i}{f^i} \Big((\mu_i - r) + \beta_i(\mu - r) \Big) + f^i_{v_i} \frac{1}{f^i} (\beta_i^2 \lambda v_t + \lambda_i \xi_{i,t}) \tag{B.5}$$

□

C Portfolio Formation and Rebalancing

The data that we use in this section is the same data described in section 3.1. Since equity option data is relatively scant both in cross-sectional and maturity dimensions, we apply a simple weighting schemes to derive option prices for the target values of moneyness and maturity. First, we calculate the implied volatilities of the options. In the next step, for each of the two maturities bracketing the target days-to-maturity of 30 days, for two values of a given target moneyness, we derive averages of IVs weighted by the reciprocal of the absolute distance from this target moneyness. Last, the two observations for IV weighted by the reciprocal of the absolute distance from the target maturity of 30 days yield the final value for IV that is subsequently inverted into the option price via the Black-Scholes formula.

In cases where we had observations with at most three days from the target maturity and/or at most 0.01 from the target moneyness, we used a single observation. In cases we had no bracketing observations for a given target, we used a nearest neighbourhood value. For the index, where we have richer data, in addition to the procedure above we also apply the practitioner's Black-Scholes via least squares and verify the sensitivity of the results to the relatively crude approach by necessity applied to equities. We choose the target moneyness ratios of 1, 1.025 and 1.05 (0.95, 0.975 and 1) for calls (puts). Standardization of option contracts is necessary to insure that the variability of the portfolio returns is only due to exposure to the risk factors. Note that the use the Black-Scholes formula to derive our target prices leaves them free of the Black-Scholes assumptions since this formula is used merely as a translation device. Once we obtain our daily option prices from cross-sections, we screen them again by rejecting observations with ATM prices below 10 cents and whose ATM IV is outside the range 5-150%. This last set of filters resulted in rejecting less than 0.1% of firm-days.

On every day and for each target moneyness ratio we set up a zero-net-cost portfolio with a long position in $\frac{1}{f^i_{v_i}}$ units of a delta hedged equity call (put) and a short position in $\frac{\beta_t^2}{f_v}$ units of a delta hedged index call (put), with the proceeds invested or borrowed at the risk free rate, r. In our calculations we approximate $f^i_{s_i}(f_s)$ and $f^i_{v_i}(f_v)$ by equity (index) option's Black-Shcoles delta and vega. Moreover, the equity beta is estimated using a rolling window of 250 days historical returns on the equity and the market. The value of the hedge portfolio at time t is:

$$\Pi_t = \frac{1}{vega_{i,t}}(f^i_t - \Delta_{i,t}S_{i,t}) - \frac{1}{vega_t}\beta^2_{i,t}(f_t - \Delta_t S_t) \tag{C.1}$$

If Π_t is positive we invest the proceeds at the risk free rate, and if it negative, we borrow this amount at the risk free rate. So after one day the gain (loss) for our zero-net-cost portfolio is:

$$\begin{aligned} G_{t+1} = \Pi_{t+1} - \Pi_t = & \frac{1}{vega_{i,t}}\Big((f^i_{t+1} - f^i_t) - \Delta_{i,t}(S_{i,t+1} - S_{i,t})\Big) \\ & - \frac{1}{vega_t}\beta^2_{i,t}\Big((f_{t+1} - f_t) - \Delta_t(S_{t+1} - S_t)\Big) - \Pi_t(\frac{r}{252}) \end{aligned} \tag{C.2}$$

We register the gain G_{t+1}, and repeat this exercise until it is done for every day in our sample. This hedge portfolio is by construction insensitive with respect to the changes in the equity price, index price and the market variance, and is only exposed to the idiosyncratic variance risk of the equity. So, the daily gains can be thought of as excess dollar return for bearing idiosyncratic volatility risk. In order to transform the excess dollar returns into percentage return and since the option price is homogenous of first degree with respect to the initial stock price, we scale the dollar returns by the initial stock price. Finally, we compound the daily portfolio returns into monthly returns.

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Table 1: **Company Names, Tickers, and the Number of Options**

		Number of Options		
Company Name	**Ticker**	**Calls**	**Puts**	**All**
S&P500 Index	**SPX**	97,355	70,934	168,289
Alcoa	**AA**	52,254	45,251	97,505
American Express	**AXP**	58,391	49,264	107,655
Boeing	**BA**	60,897	52,182	113,079
Caterpillar	**CAT**	57,977	50,036	108,013
Cisco	**CSCO**	49,533	42,555	92,088
Chevron	**CVX**	76,709	63,163	139,872
Dupont	**DD**	68,408	57,032	125,440
Disney	**DIS**	60,406	51,482	111,888
General Electric	**GE**	64,687	52,571	117,258
Home Depot	**HD**	59,296	50,778	110,074
Hewlett-Packard	**HPQ**	50,888	44,397	95,285
IBM	**IBM**	69,281	58,509	127,790
Intel	**INTC**	49,624	42,572	92,196
Johnson & Johnson	**JNJ**	84,686	67,461	152,147
JP Morgan	**JPM**	61,264	49,993	111,257
Coca Cola	**KO**	78,875	63,576	142,451
McDonald's	**MCD**	70,192	59,008	129,200
3M	**MMM**	75,277	62,499	137,776
Merck	**MRK**	67,354	55,852	123,206
Microsoft	**MSFT**	56,630	47,837	104,467
Pfizer	**PFE**	60,483	51,830	112,313
Procter & Gamble	**PG**	84,061	67,733	151,794
AT&T	**T**	74,071	58,506	132,577
United Technologies	**UTX**	70,371	58,781	129,152
V erizon	**VZ**	71,543	56,265	127,808
Walmart	**WMT**	70,668	59,539	130,207
Exxon Mobil	**XOM**	76,187	63,202	139,389

For each firm we present the name of the company, and its ticker symbol. We also report the number of Calls, Puts, and the total number of options available in our sample for each firm.

Table 2: **Option Statistics: Calls**

Ticker	Avg. IV	Min (IV)	Max (IV)	Avg. Delta	Avg. vega	Avg. DTM
SPX	0.180	0.073	0.750	0.417	198.015	80.608
AA	0.342	0.169	1.496	0.476	6.910	79.574
AXP	0.296	0.127	1.482	0.460	10.144	79.509
BA	0.293	0.161	0.896	0.461	10.666	78.889
CAT	0.309	0.160	1.033	0.466	11.417	78.849
CSCO	0.355	0.159	1.071	0.471	5.841	78.540
CVX	0.232	0.128	0.944	0.447	13.470	79.645
DD	0.259	0.123	0.923	0.459	8.852	80.528
DIS	0.283	0.069	0.959	0.466	6.720	81.597
GE	0.257	0.069	1.489	0.467	8.622	84.010
HD	0.294	0.153	1.009	0.464	7.310	80.183
HPQ	0.342	0.153	0.979	0.473	7.531	79.760
IBM	0.253	0.119	0.868	0.443	19.123	79.475
INTC	0.349	0.173	0.909	0.481	7.443	81.488
JNJ	0.199	0.097	0.708	0.438	11.388	82.155
JPM	0.293	0.112	1.489	0.461	8.993	79.048
KO	0.211	0.083	0.693	0.445	9.410	83.176
MCD	0.244	0.116	0.789	0.455	8.349	80.995
MMM	0.233	0.125	0.796	0.443	15.130	79.134
MRK	0.263	0.143	0.852	0.463	9.902	80.410
MSFT	0.290	0.122	0.879	0.475	9.099	83.745
PFE	0.272	0.142	1.010	0.471	7.145	84.019
PG	0.201	0.093	0.643	0.436	12.913	81.913
T	0.238	0.102	0.822	0.465	6.177	80.458
UTX	0.249	0.132	0.823	0.446	13.176	79.456
VZ	0.240	0.092	0.870	0.473	7.905	82.906
WMT	0.239	0.112	0.673	0.446	8.903	81.216
XOM	0.227	0.126	0.848	0.445	11.574	80.630

For each firm we report the average, minimum, and maximum implied volatility (IV) for the call options. In the last three columns we also report the average call delta, average vega, and average days-to-maturity (DTM) of the calls. The implied volatilities and the options deltas are provided by OptionMetrics, while the vega is calculated using Black-Scholes formula evaluated at the implied volatility of the options.

Table 3: **Option Statistics: Puts**

Ticker	Avg. IV	Min (IV)	Max (IV)	Avg. Delta	Avg. vega	Avg. DTM
SPX	0.208	0.089	0.784	-0.349	188.464	80.669
AA	0.354	0.174	1.484	-0.384	6.607	77.623
AXP	0.312	0.122	1.494	-0.379	9.826	78.367
BA	0.305	0.174	0.924	-0.380	10.172	78.019
CAT	0.324	0.179	1.026	-0.382	10.963	78.630
CSCO	0.365	0.163	1.096	-0.391	5.508	76.969
CVX	0.248	0.117	0.965	-0.366	12.955	79.686
DD	0.276	0.137	0.942	-0.373	8.537	80.319
DIS	0.300	0.143	0.987	-0.381	6.382	80.566
GE	0.277	0.071	1.496	-0.375	8.329	83.703
HD	0.309	0.140	1.020	-0.382	7.030	79.012
HPQ	0.352	0.164	0.920	-0.390	7.304	78.551
IBM	0.270	0.124	0.885	-0.372	18.553	79.914
INTC	0.358	0.164	0.900	-0.392	7.300	79.531
JNJ	0.220	0.096	0.768	-0.362	10.834	81.336
JPM	0.315	0.120	1.492	-0.375	8.848	79.156
KO	0.231	0.095	0.670	-0.367	9.005	82.628
MCD	0.263	0.125	0.699	-0.375	7.902	80.480
MMM	0.250	0.138	0.844	-0.367	14.682	80.075
MRK	0.279	0.091	0.881	-0.375	9.589	79.789
MSFT	0.306	0.112	0.913	-0.389	8.960	82.884
PFE	0.288	0.139	0.709	-0.383	6.984	84.079
PG	0.222	0.096	0.682	-0.361	12.427	81.336
T	0.260	0.103	0.836	-0.373	6.032	80.409
UTX	0.269	0.136	0.856	-0.372	12.667	79.524
VZ	0.262	0.109	0.896	-0.370	7.655	82.216
WMT	0.256	0.114	0.675	-0.373	8.557	80.655
XOM	0.243	0.128	0.953	-0.368	11.056	80.150

For each firm we report the average, minimum, and maximum implied volatility (IV) for the put options. In the last three columns we also report the average delta, average vega, and average days-to-maturity (DTM) of the puts. The implied volatilities and the options deltas are provided by OptionMetrics, while the vega is calculated using Black-Scholes formula evaluated at the implied volatility of the options.

Table 4: **Market and Equity Models Parameter Estimates**

Ticker	μ	κ	θ	σ	ρ	β	λ	$\tilde{\kappa}$	$\tilde{\theta}$
SPX	0.078	3.157	0.037	0.318	-0.494		-1.211	1.946	0.061
AA	0.021	1.678	0.041	0.297	-0.277	1.09	-0.872	0.806	0.086
AXP	0.116	0.409	0.049	0.201	-0.355	1.23	0.013	0.422	0.048
BA	0.072	0.480	0.062	0.084	-0.521	1.06	0.040	0.519	0.057
CAT	0.152	0.719	0.063	0.138	-0.275	1.11	-0.012	0.707	0.064
CSCO	0.116	0.417	0.107	0.165	0.100	1.08	-0.013	0.405	0.111
CVX	0.118	1.583	0.042	0.185	-0.289	0.81	0.020	1.603	0.041
DD	0.050	0.378	0.053	0.090	-0.458	0.96	0.080	0.458	0.044
DIS	0.065	0.579	0.060	0.128	-0.269	1.07	0.055	0.634	0.054
GE	0.049	0.844	0.049	0.287	-0.204	0.86	-0.229	0.615	0.067
HD	0.117	0.801	0.101	0.242	-0.121	1.06	0.289	1.090	0.074
HPQ	0.061	0.330	0.146	0.148	0.052	1.14	0.234	0.565	0.086
IBM	0.145	0.138	0.078	0.068	0.046	0.97	0.024	0.162	0.066
INTC	0.111	0.677	0.126	0.185	-0.101	0.99	0.308	0.985	0.087
JNJ	0.087	0.713	0.038	0.086	-0.481	0.60	-0.282	0.431	0.063
JPM	0.084	0.785	0.130	0.326	-0.649	0.85	0.427	1.212	0.084
KO	0.055	0.365	0.049	0.080	-0.151	0.75	-0.100	0.265	0.067
MCD	0.121	1.645	0.055	0.182	-0.511	0.55	-0.543	1.101	0.081
MMM	0.085	0.239	0.034	0.057	-0.122	0.98	0.006	0.245	0.033
MRK	0.061	0.277	0.063	0.077	-0.231	0.89	0.092	0.368	0.047
MSFT	0.110	0.268	0.066	0.077	0.060	1.04	-0.053	0.216	0.082
PFE	0.078	0.760	0.089	0.143	-0.233	0.78	0.173	0.933	0.072
PG	0.102	0.346	0.044	0.080	-0.108	0.75	0.053	0.398	0.038
T	0.066	1.026	0.049	0.173	-0.362	0.81	-0.093	0.933	0.054
UTX	0.131	0.556	0.036	0.087	0.032	1.03	-0.115	0.441	0.046
VZ	0.075	1.228	0.059	0.192	-0.602	0.70	-0.117	1.111	0.065
WMT	0.120	0.991	0.039	0.139	-0.092	0.70	-0.366	0.625	0.061
XOM	0.115	0.699	0.106	0.191	-0.370	0.75	1.145	1.845	0.040
Average	0.092	0.701	0.068	0.152	-0.240	0.91	0.006	0.707	0.064
Min	0.021	0.138	0.034	0.057	-0.649	0.551	-0.872	0.162	0.033
Max	0.152	1.678	0.146	0.326	0.100	1.225	1.145	1.845	0.111

We use OTM options over the period 1996-2011 to estimate the market and equity parameters. The estimation is based on a joint likelihood function that has a return component and an option component. The estimation of the equity model is conditional on the estimates of the market model. For the market model, μ is set to the sample average risk premium. For the equity model, μ is set equal to the intercept of the CAPM regression of the equity returns on market excess returns. Equity beta is a free parameter and is assumed constant.

Table 5: **Distributional Properties of Filtered Spot Idiosyncratic Variances**

	P-distribution				**Q-disribution**			
Ticker	**mean**	**std**	**min**	**max**	**mean**	**std**	**min**	**max**
SPX	0.0376	0.0342	0.0034	0.2608	0.0409	0.0394	0.0035	0.2956
AA	0.0942	0.0814	0.0083	0.4974	0.1078	0.1001	0.0091	0.6166
AXP	0.0750	0.0889	0.0008	0.4934	0.0711	0.0839	0.0008	0.4645
BA	0.0594	0.0280	0.0126	0.1362	0.0564	0.0264	0.0123	0.1313
CAT	0.0649	0.0323	0.0091	0.1564	0.0629	0.0308	0.0086	0.1443
CSCO	0.1311	0.1146	0.0138	0.4957	0.1292	0.1144	0.0134	0.4935
CVX	0.0396	0.0237	0.0079	0.1763	0.0384	0.0226	0.0079	0.1685
DD	0.0502	0.0305	0.0047	0.1286	0.0463	0.0281	0.0043	0.1204
DIS	0.0556	0.0383	0.0077	0.1409	0.0524	0.0360	0.0075	0.1345
GE	0.0638	0.0688	0.0016	0.4675	0.0643	0.0696	0.0016	0.4732
HD	0.0688	0.0535	0.0068	0.2583	0.0624	0.0478	0.0066	0.2350
HPQ	0.1089	0.0840	0.0105	0.3509	0.0968	0.0740	0.0102	0.3085
IBM	0.0512	0.0440	0.0050	0.1763	0.0485	0.0421	0.0049	0.1691
INTC	0.1267	0.0905	0.0168	0.4613	0.1138	0.0794	0.0159	0.4079
JNJ	0.0314	0.0200	0.0033	0.1009	0.0348	0.0230	0.0036	0.1118
JPM	0.1257	0.1356	0.0024	0.8304	0.1125	0.1160	0.0023	0.7225
KO	0.0327	0.0260	0.0036	0.1066	0.0340	0.0277	0.0036	0.1122
MCD	0.0543	0.0314	0.0067	0.1525	0.0596	0.0354	0.0071	0.1700
MMM	0.0272	0.0151	0.0076	0.0661	0.0259	0.0145	0.0074	0.0633
MRK	0.0516	0.0212	0.0127	0.1124	0.0475	0.0190	0.0119	0.1001
MSFT	0.0618	0.0439	0.0061	0.1778	0.0626	0.0450	0.0061	0.1820
PFE	0.0638	0.0340	0.0136	0.1535	0.0598	0.0315	0.0133	0.1442
PG	0.0328	0.0295	0.0037	0.1295	0.0312	0.0282	0.0036	0.1246
T	0.0551	0.0415	0.0023	0.1636	0.0550	0.0416	0.0024	0.1639
UTX	0.0342	0.0235	0.0058	0.1066	0.0348	0.0245	0.0058	0.1100
VZ	0.0555	0.0393	0.0023	0.1842	0.0560	0.0399	0.0021	0.1856
WMT	0.0519	0.0428	0.0060	0.1880	0.0564	0.0483	0.0061	0.2112
XOM	0.0481	0.0299	0.0047	0.2079	0.0381	0.0214	0.0042	0.1571

For every firm we report the mean, standard deviation, minimum, and maximum of time-series of the filtered idiosyncratic variances. The spot idiosyncratic variance are filtered from the returns, based on the optimal parameter estimates under the P and Q measures. In the top row we also report the statistics for the time-series of the market spot variance.

Table 6: **Correlation Matrix of Idiosyncratic Variances**

	AA	AXP	BA	CAT	CSCO	CVX	DD	DIS	GE	HD	HPQ	IBM	INTC	JNJ	JPM	KO	MCD	MMM	MRK	MSFT	PFE	PG	T	UTX	VZ	WMT	XOM	SPX
AA	1.00																											
AXP	0.88	1.00																										
BA	0.48	0.59	1.00																									
CAT	0.63	0.66	0.68	1.00																								
CSCO	0.04	0.15	0.65	0.18	1.00																							
CVX	0.66	0.62	0.46	0.59	0.03	1.00																						
DD	0.55	0.63	0.83	0.83	0.51	0.54	1.00																					
DIS	0.34	0.42	0.88	0.55	0.76	0.29	0.74	1.00																				
GE	0.85	0.89	0.65	0.60	0.32	0.61	0.61	0.53	1.00																			
HD	0.46	0.49	0.74	0.47	0.64	0.47	0.75	0.77	0.56	1.00																		
HPQ	0.03	0.11	0.64	0.30	0.85	0.08	0.63	0.80	0.23	0.73	1.00																	
IBM	0.01	0.12	0.61	0.36	0.78	0.14	0.70	0.70	0.20	0.71	0.89	1.00																
INTC	0.21	0.29	0.73	0.33	0.91	0.20	0.66	0.83	0.43	0.81	0.88	0.86	1.00															
JNJ	0.10	0.18	0.63	0.47	0.56	0.37	0.71	0.66	0.29	0.66	0.73	0.83	0.68	1.00														
JPM	0.88	0.91	0.61	0.59	0.19	0.69	0.60	0.49	0.93	0.58	0.15	0.15	0.36	0.26	1.00													
KO	0.16	0.26	0.66	0.54	0.65	0.32	0.82	0.69	0.28	0.74	0.82	0.92	0.77	0.88	0.26	1.00												
MCD	0.26	0.30	0.70	0.44	0.50	0.43	0.64	0.76	0.37	0.78	0.68	0.66	0.68	0.79	0.41	0.71	1.00											
MMM	0.09	0.19	0.66	0.57	0.68	0.30	0.79	0.68	0.21	0.63	0.77	0.85	0.72	0.78	0.17	0.90	0.58	1.00										
MRK	0.56	0.68	0.64	0.63	0.37	0.64	0.76	0.51	0.63	0.66	0.46	0.55	0.53	0.66	0.63	0.68	0.61	0.58	1.00									
MSFT	0.19	0.33	0.80	0.47	0.87	0.22	0.79	0.84	0.40	0.79	0.90	0.93	0.91	0.80	0.33	0.89	0.70	0.85	0.62	1.00								
PFE	0.27	0.37	0.68	0.64	0.47	0.53	0.80	0.64	0.42	0.67	0.60	0.75	0.65	0.85	0.42	0.82	0.68	0.77	0.75	0.74	1.00							
PG	0.11	0.23	0.60	0.58	0.58	0.34	0.82	0.60	0.21	0.66	0.72	0.85	0.66	0.84	0.19	0.94	0.63	0.93	0.67	0.83	0.82	1.00						
T	0.34	0.38	0.75	0.43	0.63	0.42	0.68	0.82	0.49	0.83	0.72	0.73	0.81	0.77	0.50	0.74	0.88	0.60	0.61	0.80	0.69	0.61	1.00					
UTX	0.20	0.28	0.76	0.50	0.78	0.25	0.75	0.85	0.36	0.77	0.88	0.83	0.84	0.78	0.32	0.84	0.77	0.80	0.60	0.89	0.68	0.77	0.80	1.00				
VZ	0.42	0.44	0.73	0.45	0.57	0.49	0.68	0.76	0.55	0.84	0.62	0.68	0.76	0.73	0.57	0.69	0.82	0.56	0.65	0.74	0.72	0.58	0.92	0.73	1.00			
WMT	0.18	0.25	0.62	0.55	0.61	0.38	0.80	0.68	0.28	0.79	0.79	0.89	0.75	0.87	0.26	0.95	0.76	0.88	0.67	0.86	0.82	0.93	0.76	0.84	0.73	1.00		
XOM	0.61	0.59	0.51	0.52	0.17	0.91	0.55	0.40	0.66	0.59	0.21	0.26	0.34	0.46	0.70	0.38	0.54	0.32	0.68	0.33	0.55	0.36	0.55	0.37	0.62	0.44	1.00	
SPX	0.76	0.71	0.54	0.46	0.18	0.77	0.48	0.41	0.81	0.54	0.11	0.10	0.33	0.26	0.84	0.20	0.41	0.11	0.50	0.26	0.37	0.12	0.51	0.25	0.59	0.23	0.80	1.00

We present the sample correlation matrix of the spot idiosyncratic variances under the P distribution. Presented in the last row is the pairwise correlation between the spot idiosyncratic variances and the market spot variance. The results under the Q distribution is virtually the same.

Table 7: **PCA of Implied Idiosyncratic Variance Levels**

Ticker	1st Component	2nd Component
AA	0.1407	0.4762
AXP	0.2624	0.4128
BA	0.1339	0.0320
CAT	0.1107	0.1676
CSCO	0.3199	-0.3363
CVX	0.0642	0.1247
DD	0.1494	0.0926
DIS	0.1836	-0.0227
GE	0.2323	0.2998
HD	0.2109	0.0100
HPQ	0.2347	-0.2954
IBM	0.2230	-0.1783
INTC	0.2650	-0.1442
JNJ	0.1507	-0.1072
JPM	0.2771	0.3544
KO	0.1775	-0.1058
MCD	0.1435	-0.0485
MMM	0.1114	-0.0485
MRK	0.1038	0.0509
MSFT	0.2414	-0.1407
PFE	0.1351	-0.0211
PG	0.1608	-0.0988
T	0.2238	-0.0279
UTX	0.1594	-0.0485
VZ	0.1953	-0.0095
WMT	0.2209	-0.1148
XOM	0.0751	0.0656
Average	0.1817	0.0125
Minimum	0.0642	-0.3363
Maximum	0.3199	0.4762
Variation explained	58%	23%
Correlation with the average implied idiosyncratic volatility level	99%	4.30%
Corrleation with market implied volatility level	65%	55%

We report the loadings on the first two principal components of the implied idiosyncratic variance levels obtained from the option prices. We also present the percentage of variance explained by the first two components, as well as their correlations with average implied idiosyncratic variance level of all firms, and with the implied variance levels of the market.

Table 8: **PCA of Implied Idiosyncratic Variance Moneyness Slopes**

Ticker	1st Component	2nd Component
AA	0.0876	0.0617
AXP	0.1936	0.1124
BA	0.2002	-0.0591
CAT	0.2364	0.0158
CSCO	0.1815	-0.0252
CVX	0.1805	-0.0590
DD	0.1921	0.0414
DIS	0.1934	-0.0880
GE	0.1340	0.8307
HD	0.2016	-0.1263
HPQ	0.2016	-0.2163
IBM	0.2304	-0.1086
INTC	0.1591	-0.0009
JNJ	0.1870	0.1110
JPM	0.1520	0.0657
KO	0.1856	-0.0971
MCD	0.1271	-0.0581
MMM	0.2559	-0.1304
MRK	0.1877	0.0315
MSFT	0.2277	0.0553
PFE	0.1110	0.2855
PG	0.2846	0.0278
T	0.2084	-0.0101
UTX	0.2159	-0.2536
VZ	0.1580	0.0500
WMT	0.2057	0.0435
XOM	0.1718	-0.0113
Average	0.1878	0.0181
Minimum	0.0876	-0.2536
Maximum	0.2846	0.8307
Variation explained	48%	6%
Correlation with the average implied idiosyncratic volatility moneyness slope	99%	3.40%
Corrleation with market implied volatility moneyness slope	42%	8%

We report the loadings on the first two principal components of the implied idiosyncratic variance moneyness slopes obtained from the option prices. We also present the percentage of variance explained by the first two components, as well as their correlations with average implied idiosyncratic variance moneyness slopes of all firms, and with the implied variance moneyness slope of the market.

Table 9: **PCA of Implied Idiosyncratic Variance Term Structure Slopes**

Ticker	**1st Component**	**2nd Component**
AA	0.1978	-0.4083
AXP	0.2834	-0.3145
BA	0.2173	0.0617
CAT	0.2234	-0.1185
CSCO	0.2523	0.4405
CVX	0.1471	0.0145
DD	0.1912	-0.0345
DIS	0.2364	0.0845
GE	0.1579	-0.2983
HD	0.1801	0.1234
HPQ	0.2554	0.3761
IBM	0.2317	0.0521
INTC	0.1682	-0.0250
JNJ	0.1208	0.0078
JPM	0.2145	-0.4184
KO	0.1430	-0.0096
MCD	0.1101	0.0332
MMM	0.2007	-0.0189
MRK	0.1747	0.0396
MSFT	0.2170	0.1780
PFE	0.1215	-0.0006
PG	0.1507	0.1007
T	0.1864	0.1056
UTX	0.2334	-0.1498
VZ	0.1546	0.0610
WMT	0.1282	0.0863
XOM	0.1519	0.0114
Average	0.1870	-0.0007
Minimum	0.1101	-0.4184
Maximum	0.2834	0.4405
Variation explained	61%	7%
Correlation with the average implied idiosyncratic volatility term structure slope	99%	-0.12%
Corrleation with market implied volatility term structure slope	78%	-13%

We report the loadings on the first two principal components of the implied idiosyncratic variance term structure slopes obtained from the option prices. We also present the percentage of variance explained by the first two components, as well as their correlations with average implied idiosyncratic variance term structure slopes of all firms, and with the implied variance term structure slope of the market.

Table 10: **PCA of Filtered Spot Idiosyncratic Variances**

	Physicla distribution		Risk-neutral distribution	
Ticker	**1st Component**	**2nd Component**	**1st Component**	**2nd Component**
AA	0.2395	0.3321	0.3087	-0.4391
AXP	0.2930	0.3277	0.2901	-0.3195
BA	0.1079	-0.0171	0.1070	0.0177
CAT	0.0918	0.0502	0.0908	-0.0433
CSCO	0.3541	-0.4436	0.3701	0.4550
CVX	0.0591	0.0642	0.0577	-0.0597
DD	0.1144	-0.0075	0.1113	0.0108
DIS	0.1429	-0.0720	0.1398	0.0754
GE	0.2505	0.2019	0.2658	-0.2126
HD	0.2070	-0.0619	0.1930	0.0648
HPQ	0.2562	-0.3417	0.2363	0.3092
IBM	0.1305	-0.1711	0.1331	0.1671
INTC	0.3324	-0.2861	0.3043	0.2617
JNJ	0.0570	-0.0506	0.0672	0.0614
JPM	0.4806	0.4829	0.4354	-0.4078
KO	0.0818	-0.0735	0.0903	0.0809
MCD	0.0981	-0.0490	0.1170	0.0582
MMM	0.0421	-0.0466	0.0423	0.0466
MRK	0.0721	0.0140	0.0670	-0.0095
MSFT	0.1595	-0.1389	0.1708	0.1508
PFE	0.1062	-0.0428	0.1018	0.0469
PG	0.0805	-0.0792	0.0817	0.0791
T	0.1506	-0.0670	0.1592	0.0721
UTX	0.0807	-0.0692	0.0880	0.0759
VZ	0.1436	-0.0366	0.1544	0.0403
WMT	0.1327	-0.1153	0.1598	0.1338
XOM	0.0859	0.0603	0.0611	-0.0357
Average	0.1611	-0.0236	0.1631	0.0252
Minimum	0.0421	-0.4436	0.0423	-0.4391
Maximum	0.4806	0.4829	0.4354	0.4550
Variation explained	57%	30%	55%	31%
Correlation with the average implied idiosyncratic volatility all firms	98%	-10.50%	98%	11.50%
Corrleation with market spot volatilities	68%	48%	67%	-50%

We report the loadings on the first two principal components of the spot idiosyncratic variance levels obtained from the returns. We also present the percentage of variance explained by the first two components, as well as their correlations with average spot idiosyncratic variances of all firms, and with the spot variances of the market.

Table 11: **Cross-Sectional Test of Different Asset Pricing Models**

	1	**p-val**	**2**	**p-val**	**3**	**p-val**	**4**	**p-val**	**5**	**p-val**
intercept	0.00141	(0)	0.00145	(0)	0.00137	(0)	0.0012	(0)	0.0011	(0)
Mrkt	-0.001	(0)	-0.0011	(0)	-0.00105	(0)	-0.0009	(0)	-0.0009	(0)
SMB	0.000076	(0.103)			8.60E-05	(0.079)			0.0001	(0.1248)
HML			0.0001	(0.12)	0.0001	(0.0407)			0.0001	(0.0226)
AIV							0.0001	(0.0326)	0.0001	(0.043)
Adj. R2	0.8233		0.796		0.8207		0.8347			0.8431

The table presents the factor risk premiums in the cross sectional regression of the Fama-French 25 portfolios mean excess returns on the time series coefficient estimates. We run the regression for different combination of the factors. Model 1 is the market and the SMB factor. Model 2 is the market and HML factor. Model 3 is the Fama-French model. Model 4 is the market and average idiosyncratic volatility, AIV, factor. Model 6 includes the Fama-French factors as well as the AIV factor. Presented in the table are also the P-values of the estimated risk premiums, as well as the adjusted R^2 for each model.

Table 12: **Mean Annualized Monthly Hedge Portfolio Returns: Calls**

Ticker	ATM	t-stat	OTM1	t-stat	OTM2	t-stat
SPX	-0.39	-15.00	-0.23	-9.00	0.26	1.23
AA	10.90	5.01	6.57	3.74	-4.14	-0.88
AXP	5.18	4.59	3.41	3.72	-2.16	-0.98
BA	0.32	0.66	-0.39	-0.87	-4.65	-1.87
CAT	1.65	2.50	0.43	0.79	-3.80	-1.47
CSCO	21.89	8.41	12.91	5.54	-13.90	-1.20
CVX	13.18	5.38	6.70	3.18	-11.49	-0.81
DD	-2.57	-4.37	-2.51	-4.90	-6.45	-2.55
DIS	6.21	5.84	4.11	4.56	-2.39	-0.68
GE	0.60	0.40	-0.80	-0.63	-7.10	-2.31
HD	0.85	0.82	-0.85	-0.97	-8.87	-1.81
HPQ	8.14	6.08	5.02	4.16	-3.25	-0.56
IBM	0.89	4.45	0.32	1.55	-2.69	-1.79
INTC	12.62	6.80	6.79	4.13	-13.86	-1.45
JNJ	-1.86	-5.65	-1.32	-5.22	-1.65	-1.12
JPM	5.00	2.37	4.00	2.51	0.51	0.13
KO	-1.92	-4.76	-1.21	-3.60	-2.74	-1.74
MCD	-1.39	-1.56	-1.05	-1.37	-0.58	-0.21
MMM	-1.32	-5.28	-1.23	-5.68	-2.19	-1.95
MRK	-5.78	-5.53	-2.13	-0.98	-5.23	-2.43
MSFT	0.48	0.21	-0.77	-0.45	-6.86	-1.58
PFE	-8.90	-4.31	-7.46	-4.50	-12.89	-2.27
PG	-2.24	-6.12	-1.60	-5.46	-2.58	-2.04
T	-13.83	-2.73	-11.35	-2.44	-20.12	-3.08
UTX	0.48	1.62	0.14	0.53	-1.49	-1.39
VZ	-19.71	-2.72	-15.47	-2.55	-14.33	-2.58
WMT	0.57	1.41	0.04	0.10	-3.98	-1.89
XOM	-0.57	-1.59	-0.74	-2.52	-2.69	-1.95
Mean	1.07		0.06		-5.98	
Max	21.89		12.91		0.51	
Min	-19.71		-15.47		-20.12	
No. significant	19		17		7	

The table presents the mean annualized monthly returns of the call portfolios for each firm in our sample. The portfolios are constructed and rebalanced in such a way that are only exposed to the idiosyncratic variance risk of the equity. For each firm we construct portfolios with target days-to-maturity of 30 days and three different target moneyness rations of 1, 1.025, and 1.05, denoted by ATM, OTM1, and OTM2, respectively. For each portfolio we also report the t-statistic of the null hypothesis that the mean return is zero. For each moneyness ratio we also report the minimum, maximum and mean returns, as well as the number of firms with returns significantly different from zero.

Table 13: **Mean Annualized Monthly Hedge Portfolio Returns: Puts**

Ticker	ATM	t-stat	OTM1	t-stat	OTM2	t-stat
SPX	0.25	13.32	-0.81	-20.49	-1.14	-11.98
AA	-2.28	-1.53	40.10	8.11	55.60	5.90
AXP	-2.68	-2.90	12.70	7.97	14.87	6.01
BA	1.35	3.64	3.91	6.32	-0.10	-0.10
CAT	0.08	0.14	5.58	6.32	0.58	0.33
CSCO	-11.86	-9.40	28.01	8.01	33.75	4.49
CVX	-7.09	-4.55	12.53	4.69	5.26	0.85
DD	4.17	9.21	7.46	11.63	1.72	1.52
DIS	-0.70	-0.86	9.08	7.94	4.75	2.33
GE	2.50	1.46	16.65	7.16	15.51	4.99
HD	-0.03	-0.04	3.67	3.45	-8.42	-4.77
HPQ	-1.79	-2.45	13.87	8.27	11.80	3.69
IBM	-0.18	-1.45	0.89	3.06	-1.46	-2.82
INTC	-3.71	-3.30	22.69	13.07	16.63	3.94
JNJ	2.08	6.32	0.69	1.32	-1.88	-2.12
JPM	-0.13	-0.10	18.69	8.74	14.88	3.75
KO	2.49	7.51	0.30	0.60	-4.14	-4.88
MCD	2.37	3.28	-0.40	-0.50	-9.59	-8.10
MMM	1.47	7.63	0.54	1.92	-3.22	-7.09
MRK	6.64	5.57	3.94	4.13	-4.92	-3.73
MSFT	2.39	0.88	11.44	4.29	4.61	1.86
PFE	11.83	5.99	10.79	5.26	-8.20	-3.09
PG	2.37	7.05	-0.63	-0.72	-5.52	-4.04
T	8.93	4.86	7.89	4.20	-1.58	-0.56
UTX	0.64	2.70	2.49	5.92	-1.16	-1.44
VZ	12.11	5.40	5.41	3.34	-9.48	-4.14
WMT	0.18	0.72	1.57	2.09	-3.65	-2.80
XOM	1.89	6.54	2.20	5.26	-3.46	-4.24
Mean	1.22		8.97		4.19	
Max	12.11		40.10		55.60	
Min	-11.86		-0.63		-9.59	
No. significant	18		22		20	

The table presents the mean annualized monthly returns of the put portfolios for each firm in our sample. The portfolios are constructed and rebalanced in such a way that are only exposed to the idiosyncratic variance risk of the equity. For each firm we construct portfolios with target days-to-maturity of 30 days and three different target moneyness rations of 1, 0.975, and 0.95, denoted by ATM, OTM1, and OTM2, respectively. For each portfolio we also report the t-statistic of the null hypothesis that the mean return is zero. For each moneyness ratio we also report the minimum, maximum and mean returns, as well as the number of firms with returns significantly different from zero.

Table 14: **Idiosyncratic Variance Risk Premiums**

Ticker	Mean (%)	std (%)	Skewness	Kurtosis	t-stat
SPX	-0.48	0.65	-3.675	18.687	-46.34
AA	-1.63	2.17	-3.204	14.014	-47.03
AXP	0.38	0.52	2.843	12.005	46.17
BA	0.30	0.19	0.513	2.595	97.83
CAT	0.19	0.25	3.064	13.951	48.49
CSCO	0.18	0.25	2.447	10.819	45.06
CVX	0.12	0.14	2.759	11.447	53.82
DD	0.39	0.30	1.088	4.296	81.52
DIS	0.33	0.30	1.334	4.819	67.97
GE	-0.10	0.22	-3.757	21.760	-28.17
HD	0.69	0.68	1.328	3.834	63.26
HPQ	1.29	1.23	1.313	4.173	65.45
IBM	0.27	0.24	1.050	3.996	69.55
INTC	1.39	1.27	1.273	3.861	68.98
JNJ	-0.37	0.42	-2.181	9.387	-55.86
JPM	1.47	2.23	3.252	14.382	41.32
KO	-0.14	0.30	-3.273	16.391	-30.26
MCD	-0.62	0.50	-1.267	4.292	-77.25
MMM	0.13	0.09	0.622	2.168	92.34
MRK	0.42	0.31	1.477	5.385	85.48
MSFT	-0.09	0.31	-2.090	10.644	-17.41
PFE	0.43	0.32	1.071	3.770	83.00
PG	0.17	0.15	0.813	2.552	68.90
T	-0.02	0.10	0.032	7.750	-9.92
UTX	-0.08	0.18	-1.890	8.385	-26.36
VZ	-0.08	0.11	-1.599	6.594	-42.97
WMT	-0.51	0.63	-1.721	5.080	-50.95
XOM	1.15	1.10	2.706	11.110	65.71

For each firm we present the annualized first four moments of the time-series of idiosyncratic variance risk premium. The idiosyncratic variance risk premium is calculated as the difference between the excepted integrated idiosyncratic variance under P and Q distributions. In the last column we report the t-stat for the null hypothesis that the average idiosyncratic variance risk premium is zero.

Table 15: **Fama-French Regression of the Idiosyncratic Variance Risk Premiums**

Ticker	a	t-stat	b^m	t-stat	b^{smb}	t-stat	b^{hml}	t-stat	b^{mom}	t-stat	R^2 (%)
AA	-0.016	-47.418	0.049	1.746	-0.059	-1.074	0.072	1.316	0.233	6.330	1.03
AXP	0.004	46.532	-0.007	-1.123	0.019	1.419	-0.024	-1.841	-0.053	-6.053	0.98
BA	0.003	97.938	-0.001	-0.472	0.008	1.677	-0.002	-0.477	-0.012	-3.557	0.39
CAT	0.002	48.857	-0.002	-0.728	0.011	1.684	-0.011	-1.791	-0.026	-6.105	1.03
CSCO	0.002	45.470	-0.004	-1.408	0.010	1.645	-0.017	-2.760	-0.026	-6.282	1.08
CVX	0.001	54.253	-0.004	-2.027	0.002	0.638	-0.012	-3.507	-0.014	-5.815	0.94
DD	0.004	81.669	-0.001	-0.280	0.003	0.411	-0.004	-0.499	-0.020	-3.968	0.44
DIS	0.003	68.209	-0.003	-0.766	0.013	1.687	-0.010	-1.345	-0.024	-4.642	0.61
GE	-0.001	-28.374	0.003	0.945	0.003	0.445	-0.002	-0.363	0.019	4.923	0.75
HD	0.007	63.420	-0.017	-1.916	0.006	0.374	-0.015	-0.854	-0.044	-3.822	0.39
HPQ	0.013	65.314	-0.001	-0.048	0.019	0.614	0.043	1.369	-0.005	-0.223	0.06
IBM	0.003	69.428	0.000	-0.007	0.000	-0.044	0.007	1.114	-0.002	-0.542	0.06
INTC	0.014	68.953	-0.022	-1.345	0.034	1.042	0.020	0.616	-0.037	-1.711	0.15
JNJ	-0.004	-55.748	-0.003	-0.643	0.019	1.746	-0.012	-1.114	-0.005	-0.761	0.12
JPM	0.015	41.687	-0.050	-1.756	0.040	0.708	-0.126	-2.240	-0.231	-6.096	0.95
KO	-0.001	-30.142	-0.003	-0.732	0.014	1.817	-0.016	-2.200	-0.010	-1.958	0.26
MCD	-0.006	-77.139	0.003	0.489	0.003	0.245	-0.001	-0.045	0.006	0.716	0.02
MMM	0.001	92.270	0.000	0.308	0.004	1.675	0.002	0.854	-0.003	-1.855	0.21
MRK	0.004	85.590	-0.002	-0.555	0.000	-0.036	0.001	0.163	-0.019	-3.565	0.38
MSFT	-0.001	-17.280	-0.003	-0.719	0.016	2.039	-0.016	-2.057	-0.017	-3.267	0.41
PFE	0.004	83.118	-0.003	-0.784	-0.004	-0.427	-0.012	-1.419	-0.018	-3.333	0.30
PG	0.002	68.832	0.000	-0.102	-0.003	-0.681	0.004	1.056	-0.003	-1.253	0.12
T	0.000	-9.744	-0.001	-0.646	0.003	1.359	-0.007	-2.869	-0.007	-4.124	0.57
UTX	-0.001	-26.210	-0.001	-0.360	0.005	1.071	-0.013	-2.830	-0.010	-3.174	0.41
VZ	-0.001	-42.881	0.001	0.661	0.004	1.571	-0.005	-1.809	-0.003	-1.842	0.26
WMT	-0.005	-50.812	0.000	-0.031	0.010	0.652	-0.033	-2.092	-0.015	-1.440	0.16
XOM	0.012	65.989	-0.031	-2.214	-0.037	-1.344	-0.075	-2.692	-0.075	-4.014	0.53

For every firm we run a time-series regression of the idiosyncratic variance risk premium on the market excess return ($r_t^m - r_t$), Fama-French factors (r^{SMB} and r^{HML}) as well as the momentum factor (r^{mom}), according to the following equations:

$$IVRP_{i,t} = a_i + b_i^m \cdot (r_t^m - r_t) + b_i^{smb} \cdot r_t^{SMB} + b_i^{hml} \cdot r_t^{HML} + b_i^{mom} \cdot r_t^{mom} + \varepsilon_{i,t}$$

We report the regression coefficients, t-stats, and the R^2 of the regression.

Table 16: **Explanatory Power of $MVRP$ for $IVRP$**

Ticker	**a**	**t-stat**	b^m	**t-stat**	R^2 (%)
AA	-0.004	-15.305	2.448	68.301	54.35
AXP	0.001	15.893	-0.543	-58.968	47.01
BA	0.002	69.258	-0.146	-35.452	24.28
CAT	0.001	18.929	-0.252	-56.188	44.62
CSCO	0.000	10.205	-0.305	-87.259	66.02
CVX	0.000	22.477	-0.193	-134.361	82.16
DD	0.003	54.028	-0.238	-38.009	26.93
DIS	0.002	40.447	-0.274	-45.610	34.68
GE	0.000	-9.268	0.128	25.516	14.25
HD	0.004	34.883	-0.698	-56.188	44.62
HPQ	0.011	47.068	-0.313	-10.537	2.76
IBM	0.002	49.928	-0.067	-11.636	3.34
INTC	0.010	44.277	-0.769	-27.142	15.82
JNJ	-0.004	-45.858	-0.016	-1.599	0.07
JPM	0.002	6.571	-2.665	-78.404	61.07
KO	-0.002	-30.189	-0.067	-9.406	2.21
MCD	-0.005	-54.568	0.190	15.964	6.11
MMM	0.001	65.639	-0.043	-21.647	10.68
MRK	0.003	57.821	-0.237	-36.155	25.01
MSFT	-0.002	-25.803	-0.138	-18.814	8.28
PFE	0.003	55.357	-0.268	-40.224	29.22
PG	0.001	45.949	-0.074	-20.875	10.01
T	-0.001	-29.710	-0.075	-34.628	23.43
UTX	-0.001	-30.682	-0.064	-14.963	5.40
VZ	-0.001	-37.475	-0.013	-4.771	0.58
WMT	-0.005	-39.185	0.047	3.046	0.24
XOM	0.005	37.687	-1.273	-73.038	57.65

For each firm we run a time-series regression of the idiosyncratic variance risk premium on the market variance risk premium, according to the following equation:

$$IVRP_{i,t} = a_i + b_i \cdot MVRP_t + \varepsilon_{i,t}$$

We report the regression coefficients and the associated t-stats, along with the regression R^2.

Table 17: **Explanatory Power of $AVRP$ and F_{orth} for $IVRP$**

Ticker	a	t-stat	b^{AIVRP}	t-stat	b^{orth}	t-stat	R^2 (%)
AA	0.001	3.049	-8.238	-74.027	1.439	29.341	61.81
AXP	-0.001	-15.524	2.291	102.929	-0.093	-9.436	73.17
BA	0.001	56.727	0.861	113.724	0.095	28.534	77.82
CAT	0.000	-6.436	1.017	83.419	-0.065	-12.107	64.46
CSCO	0.000	-14.455	1.038	103.983	-0.173	-39.429	75.94
CVX	0.000	-7.213	0.608	155.640	-0.133	-77.314	88.52
DD	0.001	31.299	1.288	95.511	0.099	16.657	70.58
DIS	0.000	9.501	1.461	165.524	0.103	26.512	87.76
GE	0.000	11.882	-0.709	-49.298	-0.060	-9.533	39.15
HD	0.000	5.534	3.082	110.011	-0.051	-4.156	75.57
HPQ	0.005	24.568	3.737	51.893	1.132	35.678	50.30
IBM	0.001	28.054	0.720	49.302	0.202	31.427	46.59
INTC	0.003	19.145	5.062	83.300	0.760	28.400	66.41
JNJ	-0.003	-28.704	-0.540	-17.055	-0.301	-21.581	16.19
JPM	-0.006	-27.189	10.080	121.447	-1.021	-27.909	79.85
KO	-0.001	-17.532	-0.130	-5.606	-0.213	-20.959	10.73
MCD	-0.003	-34.296	-1.338	-38.270	-0.229	-14.899	30.09
MMM	0.001	45.379	0.298	60.871	0.051	23.510	52.08
MRK	0.002	35.471	1.232	75.821	0.074	10.380	59.92
MSFT	-0.001	-17.039	0.150	6.130	-0.235	-21.843	11.61
PFE	0.001	32.376	1.338	82.768	0.056	7.918	63.83
PG	0.001	22.912	0.524	59.050	0.092	23.479	50.76
T	0.000	-22.412	0.145	19.914	-0.097	-30.186	25.03
UTX	-0.001	-17.900	-0.025	-1.853	-0.155	-25.583	14.38
VZ	-0.001	-22.442	-0.101	-11.740	-0.079	-20.669	12.60
WMT	-0.003	-22.409	-0.990	-20.644	-0.380	-17.997	16.07
XOM	0.003	18.054	4.141	75.481	-0.818	-33.851	63.59

We first regress the market variance risk premium on the average variance risk premiums of all firms to get the component,F_{RP}^{orth}, of the market variance risk premium that is orthogonal to the average variance risk premium. Then we regress the idiosyncratic variance risk premium of each firm on the average variance risk premium,$AVRP$, and the component of the market variance risk premium orthogonal to it.

$$IVRP_{i,t} = a_i + b_{AIVRP,i} \cdot AIVRP_t + b_{orth,i} \cdot F_{RP}^{orth} + \varepsilon_{i,t}$$

We report the regression coefficients and the associated t-stats, along with the regression R^2.

Table 18: **Estimated Unconditional Betas**

Ticker	Joint	OLS	P	Q
AA	1.09	1.30	1.41	0.99
AXP	1.23	1.45	1.08	0.99
BA	1.06	0.93	1.11	0.98
CAT	1.11	1.08	1.28	0.99
CSCO	1.08	1.41	1.15	0.97
CVX	0.81	0.81	0.94	0.91
DD	0.96	0.99	1.20	1.00
DIS	1.07	1.05	1.05	1.00
GE	0.86	1.19	1.03	1.00
HD	1.06	1.09	1.17	0.99
HPQ	1.14	1.13	1.01	0.95
IBM	0.97	0.91	0.73	0.91
INTC	0.99	1.32	1.24	0.98
JNJ	0.60	0.57	0.48	0.67
JPM	0.85	1.55	1.12	1.01
KO	0.75	0.58	0.66	0.75
MCD	0.55	0.60	0.71	0.74
MMM	0.98	0.77	0.85	0.90
MRK	0.89	0.77	0.74	0.91
MSFT	1.04	1.09	0.90	1.00
PFE	0.78	0.81	0.93	0.95
PG	0.75	0.57	0.57	0.70
T	0.81	0.79	0.77	0.83
UTX	1.03	0.95	1.04	0.96
VZ	0.70	0.74	0.74	0.81
WMT	0.70	0.75	0.73	0.70
XOM	0.75	0.80	0.94	0.92
Average	0.91	0.96	0.95	0.91
Min	0.55	0.57	0.48	0.67
Max	1.23	1.55	1.41	1.01

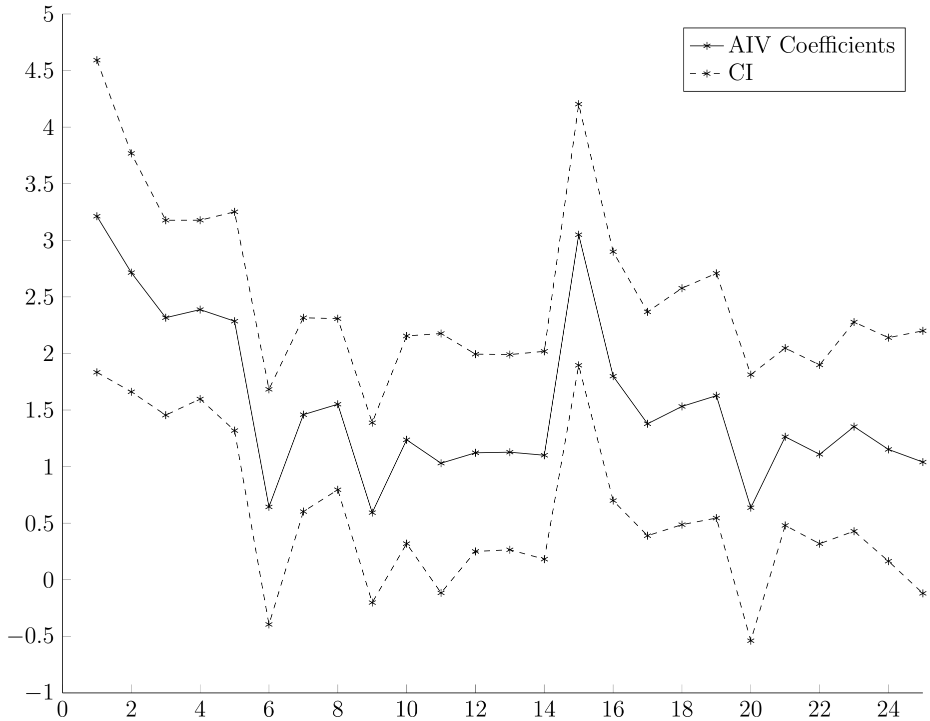
The first column reports the beta estimates based on the joint likelihood function. The second column presents the OLS beta estimate over the whole sample. The betas in the third column are estimated by fitting the equity return dynamics (2.2) to equity returns under the P measure, while the betas in the last column are estimated by fitting the model option price to the observed option prices under the Q measure.

Table 19: **Conditional Beta Estimates**

	Conditional beta				
Ticker	**mean**	**std**	**min**	**max**	**unconditional beta**
AA	1.26	0.34	0.47	2.94	1.09
AXP	1.44	0.37	0.44	3.86	1.23
BA	1.26	0.35	0.24	2.07	1.06
CAT	1.33	0.36	0.22	2.50	1.11
CSCO	1.62	0.52	0.00	3.14	1.08
CVX	0.95	0.27	0.25	1.60	0.81
DD	1.12	0.33	0.22	2.09	0.96
DIS	1.25	0.37	0.20	2.12	1.07
GE	1.07	0.38	0.00	2.34	0.86
HD	1.25	0.32	0.15	2.40	1.06
HPQ	1.42	0.39	0.01	2.78	1.14
IBM	1.29	0.32	0.01	2.22	0.97
INTC	1.36	0.47	0.10	2.72	0.99
JNJ	0.90	0.29	0.23	1.81	0.60
JPM	0.51	0.39	0.00	2.64	0.85
KO	1.01	0.34	0.08	1.99	0.75
MCD	0.49	0.22	0.00	1.70	0.55
MMM	1.10	0.29	0.04	1.83	0.98
MRK	1.08	0.42	0.09	1.96	0.89
MSFT	1.20	0.62	0.00	3.20	1.04
PFE	0.99	0.40	0.17	2.20	0.78
PG	0.98	0.31	0.05	2.00	0.75
T	0.81	0.40	0.00	2.36	0.81
UTX	1.22	0.24	0.12	1.95	1.03
VZ	0.65	0.31	0.00	2.02	0.70
WMT	1.07	0.37	0.09	2.41	0.70
XOM	0.95	0.22	0.02	1.49	0.75

We estimate conditional betas on a daily frequency for every firm. These estimates are based on the estimated market and equity models structural parameters, and the estimated spot market variance, conditional on the option prices observed on each day. We present the mean, standard deviation, minimum, and maximum of the time-series of the conditional betas. In the last column we also report our estimate of the unconditional beta baed on the joint likelihood estimation for comparison.

Figure 1: **Time-Series Coefficients of the AIV Factor**



For each of the 25 Fama-French portfolios we run the time series regression (4.4) of the portfolio's excess return onto the Fama-French factors and the average idiosyncratic volatility factor (AIV). The solid line plots the coefficient estimates, b^{AIV}, and the dashed lines represent the 95% confidence intervals.